Modified S-Probe Circuit Element For Stability Analysis

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Abstract - **The S-probe stability analysis method was originally introduced by Wang, Jones, and Nelson [1] as a noninvasive technique valid for circuits with feedback under arbitrary source and load conditions. Unfortunately, the S-probe circuit elements that are introduced into the network under investigation do not correctly calculate the impedances necessary to assess stability. The end result of this error is that the method tends to predict false instabilities, and in general does not agree with other stability methods. In this letter, the error in the S-probe circuit element is corrected, and the modified method is shown to be in agreement with the loop gain technique for an oscillator circuit example.**

I. Introduction

The S-probe stability analysis method described in [1] is a noninvasive technique valid for circuits with feedback under arbitrary source and load conditions. The primary advantage of the S-probe analysis over loop gain techniques [2,3,4] is that the method is noninvasive, and does not require the circuit to be broken in order to calculate a loop gain or return ratio. The necessity to break the circuit makes it difficult to monitor the stability properties during the design and optimization phase. Unfortunately, the S-probe circuit elements introduced in [1] do not correctly extract the network impedances required to check against the oscillation condition. The presence of this error can cause the method to predict false instabilities which may lead the circuit designer to overstablize the network sacrificing performance.

II. The S-probe Method

In the S-probe technique, near lossless circuit elements called S-probes are placed at the input and the output of the active networks as shown for the FET oscillator circuit in Figure 1.

The function of the S-probe circuit is to extract the terminating impedances Z_S, Z_{IN}, Z_{OUT} and Z_L present on either side of the elements. From these impedances reflection coefficients are calculated and checked against the oscillation conditions for a two-port network [5],

$$\Gamma_S \Gamma_{IN} = \left(\frac{Z_S - 50}{Z_S + 50}\right)\left(\frac{Z_{IN} - 50}{Z_{IN} + 50}\right) = 1 \tag{1}$$

$$\Gamma_{OUT} \Gamma_L = \left(\frac{Z_{OUT} - 50}{Z_{OUT} + 50}\right)\left(\frac{Z_L - 50}{Z_L + 50}\right) = 1 \tag{2}$$

Network stability may be assessed by observing if the polar plots versus frequency of the reflection coefficient products for given source and load terminations encircle the critical point (1,0). A more convenient monitor of circuit stability is to plot the real part of the reflection coefficient products. The stability criterion is that a network is stable if both stability indices S_1 and S_2 simultaneously satisfy the conditions,

$$S_1 = \text{Re}\{\Gamma_S \, \Gamma_{IN}\} < 1 \tag{3}$$

$$S_2 = \text{Re}\{\Gamma_{OUT} \, \Gamma_L\} < 1 \tag{4}$$

If the above conditions hold, the network can not possibly satisfy the oscillation condition. Under some conditions one stability index may be greater than one while the other is less one. In this case one must refer to the polar plots to determine network stability. The S-probe approach is to analyze the circuit over the entire frequency range that the active device model is considered accurate under worst case source and load conditions, and monitor the stability indices to insure stability.

III. Calculation of Network Impedances

Before deriving a new S-probe circuit, the error in original element will be revealed. For the S-probe circuit listed in [1] the network impedances Z_{IN} and Z_L are determined by injecting voltage v_1 at the input port and computing v_2, i_l and i_2 as shown in the shunt feedback example in Figure 2. Admittance matrix [Y] represents an amplifier and matrix [Y^F] a feedback network. These impedances are then,

$$Y_{IN} = \frac{1}{Z_{IN}} = \frac{i_1}{v_1} = y_{11} - y_{12}\left[\frac{Z_2\left(y_{21}+y_{21}^F\right)}{1+Z_2\left(y_{22}+y_{22}^F\right)}\right] \tag{5}$$

$$Y_L = \frac{1}{Z_L} = \frac{-i_2}{v_2} = \frac{1}{Z_2} + y_{22}^F - y_{21}^F\left[\frac{1+Z_2\left(y_{22}+y_{22}^F\right)}{Z_2\left(y_{21}+y_{21}^F\right)}\right] \tag{6}$$

Some observations may be made from these results. First, source impedance Z_l does not appear in the Z_L expression which does not physically make sense. Second, as the amplifier transconductance y_{2l} approaches zero,

$$Y_L \to -y_{22} \tag{7}$$

This is obviously incorrect, and it is readily apparent why false instabilities occurred when calculating impedance in this manner. Third, for the case of no feedback the expressions produce the correct results for Z_{IN} and Z_L which may have misled the authors in [1]. The correct way to extract the required network impedances is to inject the voltage signal at the point in the circuit where the impedances are being calculated. For example, to compute Z_{OUT} and Z_L voltage v_2 is the injected signal and the current flowing to the left i_2 and the current flowing to the right $i_L + i_2^F$ are calculated. The difference between the left flowing and right flowing current is sourced or sunk by the fictitious ideal voltage source supplying voltage v_2. Impedances Z_S and Z_{IN} are found by injecting voltage v_1 and repeating the calculation for left flowing current $i_S + i_l^F$ and right flowing current i_l. The results for the circuit in Figure 2 are,

$$Y_S = \frac{1}{Z_S} = \frac{1}{Z_1} + y_{11}^F - \frac{Z_2 y_{12}^F\left(y_{21} + y_{21}^F\right)}{1 + Z_2\left(y_{22} + y_{22}^F\right)} \tag{8}$$

$$Y_{IN} = \frac{1}{Z_{IN}} = y_{11} - \frac{Z_2 y_{12}\left(y_{21} + y_{21}^F\right)}{1 + Z_2\left(y_{22} + y_{22}^F\right)} \tag{9}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{Z_2} + y_{22}^F - \frac{Z_1 y_{21}^F\left(y_{12} + y_{12}^F\right)}{1 + Z_1\left(y_{11} + y_{11}^F\right)} \tag{10}$$

$$Y_{OUT} = \frac{1}{Z_{OUT}} = y_{22} - \frac{Z_1 y_{21}\left(y_{12} + y_{12}^F\right)}{1 + Z_1\left(y_{11} + y_{11}^F\right)} \tag{11}$$

Note that these expressions reduce as expected when the feedback goes to zero and when the amplifier is removed..

IV. The S-Probe Circuit Element

In order for the S-probe technique to be implemented on commercially available microwave circuit simulators, a near lossless circuit element needs to be developed to perform the impedance calculations detailed above. The modified S-probe circuit element is a six port network and is shown in Figure 3.

Ports 1 and 2 are connected to the circuit under investigation which is represented by the terminating impedances Z_1 and Z_2. These impedances are extracted from the transmission characteristics from ports 5 and 6 to ports 3 and 4. The resistors R_S and R_V are made arbitrarily large such that they do not load the circuit and therefore may be neglected in the analysis. The series resistor R must be nonzero, however it can be made small enough such that the S-probe circuit may be considered lossless. Under these assumptions, if voltage V_6 is applied to port 6, producing voltage V_A at node A, the S-parameters S_{36} and S_{46} are equal to,

$$S_{36} = \frac{-2M_3RV_A}{V_6(Z_2+R)} \qquad S_{46} = \frac{-2M_4Z_2V_A}{V_6(Z_2+R)} \tag{12}$$

Similarly, applying voltage V_5 to port 5 produces voltage V_B at node B, and the S-parameters S_{35} and S_{45} are,

$$S_{35} = \frac{2M_3RV_B}{V_5(Z_1+R)} \qquad S_{45} = \frac{-2M_4V_B}{V_5} \tag{13}$$

The ratios S_{46}/S_{36} and S_{45}/S_{35} are equal to,

$$\frac{S_{46}}{S_{36}} = \frac{M_4Z_2}{M_3R} \qquad \frac{S_{45}}{S_{35}} = \frac{-M_4(Z_1+R)}{M_3R} \tag{14}$$

To extract the terminating impedances, the VCVS gains are set to satisfy the relation.

$$\frac{M_4}{M_3R} = 1 \tag{15}$$

If R is set small enough to be negligible, the terminating impedances at port 1 and port 2 of the S-probe circuit are recovered by evaluating the following.

$$Z_1 = \frac{-S_{45}}{S_{35}} \qquad Z_2 = \frac{S_{46}}{S_{36}} \tag{16}$$

V. S-Probe Circuit Example: FET Oscillator

Shown in Figure 1 is a FET feedback oscillator circuit which is loaded to satisfy the oscillation condition at approximately 10GHz. A polar plot of the reflection coefficient products and the result of a loop gain analysis are shown in Figure 4. The resulting stability indices and the loop gain magnitude and phase are plotted in Figure 5. The loop gain analysis predicts a loop gain of unity with zero phase at 10GHz which just meets the Nyquist criterion for an unstable circuit. The S-probe analysis predicts the instability at the same frequency as evident from the polar plots of $\Gamma_S \Gamma_{IN}$ and $\Gamma_{OUT} \Gamma_L$ passing through the point (1,0), and the stability indices S_1 and S_2 both being equal to unity.

VI. Conclusion

The S-probe stability analysis method presented in [1] is a noninvasive technique valid for circuits with feedback under arbitrary source and load conditions. Unfortunately, the S-probe circuit elements used in [1] do not correctly extract the network impedances which result in disagreement with loop gain methods and in some cases false instabilities. A modified S-probe circuit element has been presented that correctly extracts the impedances required to implement the S-probe stability analysis technique. Using the modified S-probe circuit elements, stability analysis was demonstrated for an oscillator example and was shown to agree with loop gain analysis.

References

[1] K. Wang, M. Jones, and S. Nelson, "A New Cost-Effective, 4-Gamma Method for Evaluating Multi-Stage Amplifier Stability", IEEE MTT-S Digest, pp. 829-832, 1992.

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[3] A. Platzker, W. Struble, and K. Hetzler, "Instabilities Diagnosis and The Role of K in Microwave Circuits", IEEE MTT-S Digest, pp. 1185-1188, 1993.

[4] W. Struble and A. Platzker, "A Rigorous Yet Simple Method For Determining Stability of Linear N-Port Networks", GaAs IC Symposium, pp. 251-254, 1993.

[5] Gonzalez, G. *Microwave Transistor Amplifiers, Analysis and Design.* Englewood Cliffs, New Jersey: Prentice Hall Inc., 1984.

Figure Captions

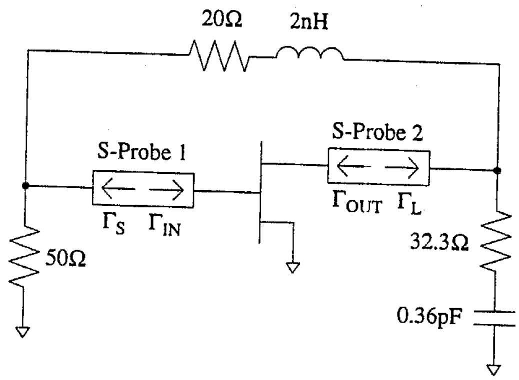
Figure 1. FET oscillator circuit

Figure 2. Shunt feedback example

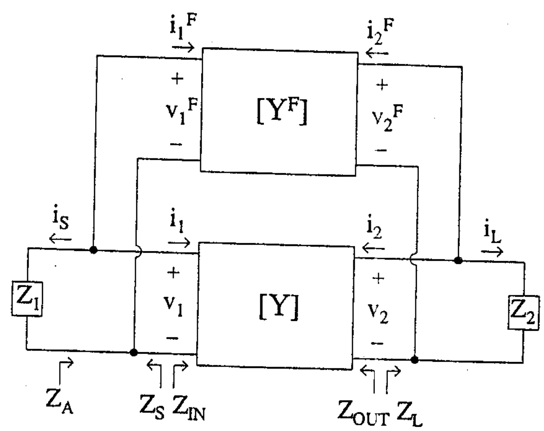
Figure 3. The modified S-probe circuit element

Figure 4. $\Gamma_S\,\Gamma_{IN}$, $\Gamma_{OUT}\,\Gamma_L$ and loop gain for the oscillator circuit

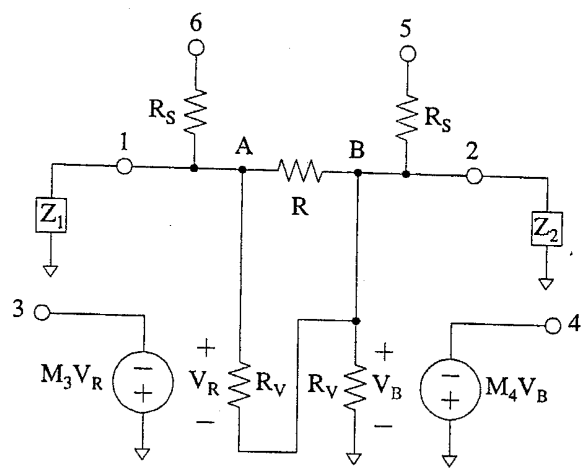
Figure 5. Stability indices and loop gain for the FET oscillator



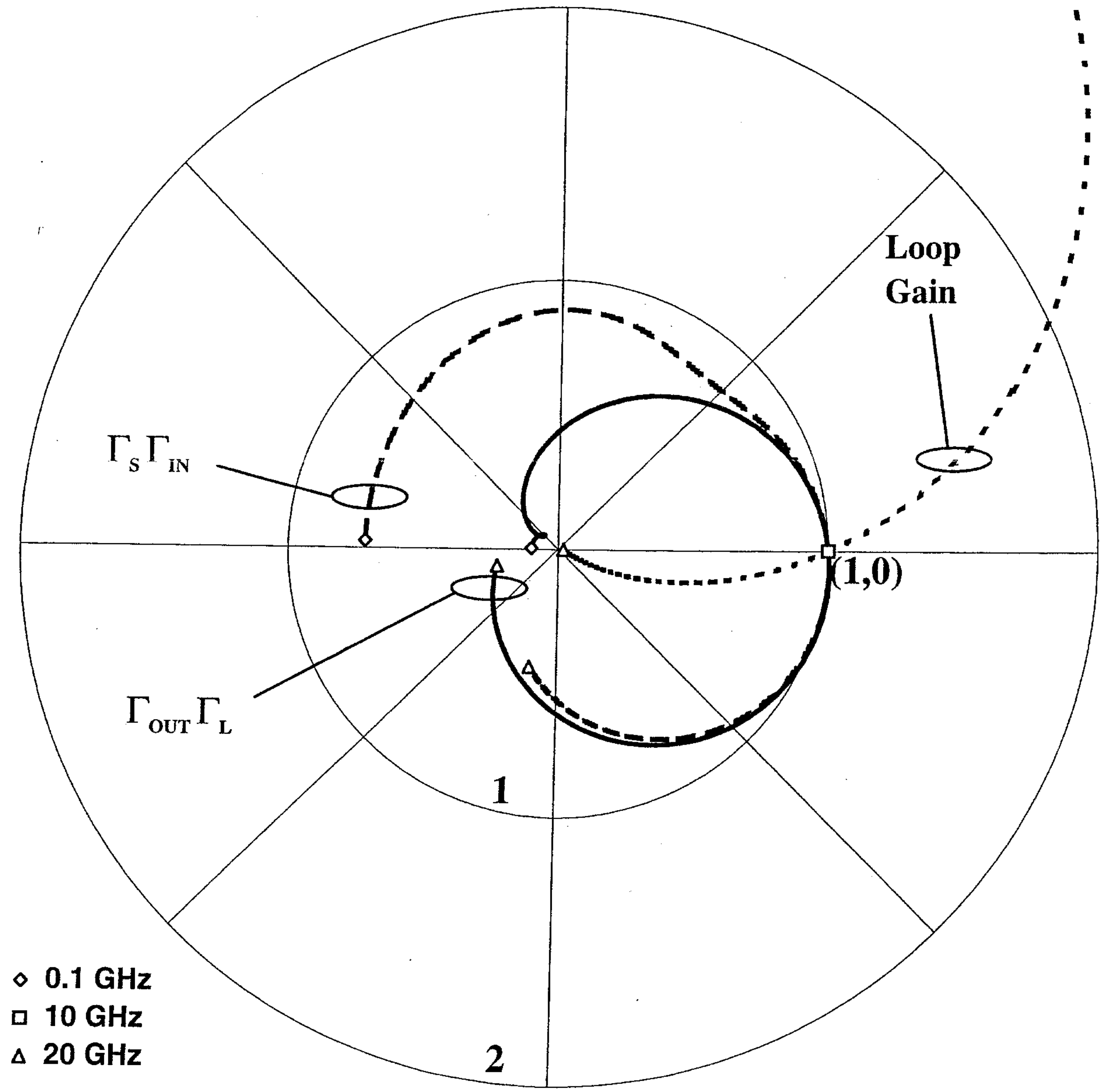
20Ω
2nH
S-Probe 1
S-Probe 2
Γ_S Γ_IN
Γ_OUT Γ_L
50Ω
32.3Ω
0.36pF

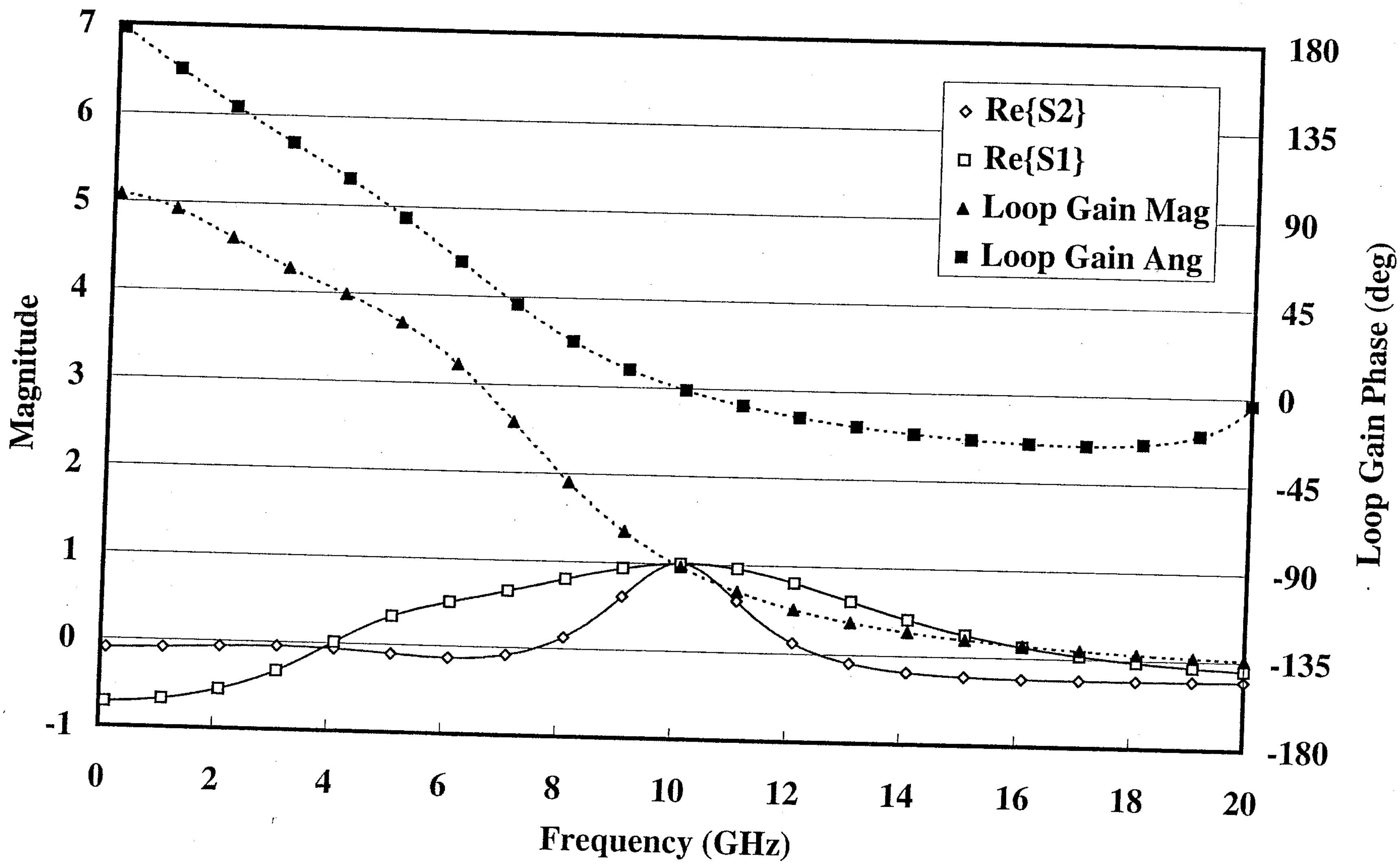


i1F
i2F
v1F
[YF]
v2F
iS
i1
i2
iL
Z1
v1
[Y]
v2
Z2
ZA
ZS ZIN
ZOUT ZL



6
5
R_S
R_S
1
A
B
2
R
Z_1
Z_2
3
4
M_3V_R
V_R
R_V
R_V
V_B
M_4V_B


Loop
Gain
$\Gamma_S \Gamma_{IN}$
$\Gamma_{OUT} \Gamma_L$
(1,0)
1
2
0.1 GHz
10 GHz
20 GHz


Re{S2}
Re{S1}
Loop Gain Mag
Loop Gain Ang
Magnitude
Loop Gain Phase (deg)
Frequency (GHz)