3) The 75 percent BW element design over a quarter-hemisphere maximum VSWR value of 16, and consequently, to be practical, will require an elaborate matching network.

4) It is apparent that, with a careful aperture design, a dual ridge rectangular waveguide offers a practical solution for a wide-band phased array element. This conclusion is confirmed by the results of [7].

5) The present study does not consider the question of polarization. A minor addition to the computer program will yield the axial ratios and the tilt angles.

REFERENCES


The Design of Small Slot Arrays

ROBERT S. ELLIOTT, FELLOW, IEEE, AND L. A. KURTZ

Abstract—The differences in mutual coupling for a central slot and a peripheral slot cannot be ignored in small arrays if good patterns and impedance are to be obtained. A theory has been developed whereby the length and offset of every slot in the array can be determined, in the presence of mutual coupling, for a specified aperture distribution and impedance match. The theory enlarges on Stevenson’s method, and uses a modified form of Booker’s relation based on Babinet’s principle to treat nonresonant longitudinal shunt slots in the broad wall of a rectangular waveguide. A general relation between slot voltage and mode is developed, and then formulas are derived for the active, self-, and mutual admittances among slots. These formulas result in a design procedure. Analogous treatments of inclined series slots in a rectangular guide and of strip-line-fed slots are possible. Comparison between various experiments and the theory is presented. Tests of the theory include the resonant length of a zero offset slot, resonant conductance versus offset and resonant conductance versus frequency for a single slot, and self- and mutual admittances for two staggered slots. The design and performance of a two-by-four longitudinal slot array is also described.

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THEORY

Consider the module consisting of the solid lines shown in Fig. 1. This is a section of rectangular waveguide λ₀/2 long containing a longitudinal slot of length 2l and displacement x cut in its upper broad wall. One- and two-dimensional slot arrays can be constructed by placing such modules in tandem and parallel positions.

The module of Fig. 1 is a two-port device, the ports being at z = ±λ₀/4 if the origin is taken in the waveguide cross section which bisects the slot. But no loss in generality occurs if the ports are taken at the positions z = ±λ₀, shown dotted in Fig. 1, because relations between the two sets of ports involve simple known linear transformations. It is convenient to choose the ports at z = ±λ₀; with this convention adopted, the equivalent circuit for the nth module1 is as shown in Fig. 2.

This equivalent circuit is subject to the following interpretation. It is assumed that only the dominant TE₁₀ mode can propagate in the waveguide. This mode is represented by the voltage/current pair Vₙ, Iₙ at the input port (z = −λ₀). A load

1 For notational simplicity the single index n is used to identify this module, but it is important to remember that these modules can be arranged to form either a linear array or a planar array. Double subscript notation could be used in the latter case.
The admittance $Y_{n}^{L}$ is placed at the output port ($z = +\lambda_{g}$). This admittance (transformed through $3\lambda_{g}/4$) could represent what the $n$th module "sees" looking down its branch line at all the modules beyond, or it could be an appropriate termination, wherein the admittance among the modules.

In words, (4) states that the active admittance at the terminals of the $n$th module equals the self-admittance of the $n$th slot plus a term which accounts for mutual coupling. This latter term is a summation which involves not only the mutual admittances between ports, but also the relative voltages at the different ports. As the analysis develops further, it will be seen that $Y_{n}^{A}$ is decisive in determining the amplitude and phase of the electric field in the $n$th slot. Since the latter is dictated by the desired radiation pattern, $Y_{n}^{A}$ becomes the focal point of array design.

It is well known that the scattering off a shunt element is symmetrical and given by

$$B = C = -\frac{1}{2} \frac{Y_{n}^{A}}{G_{0}} V_{n}$$

in which $B$ and $C$ are the amplitudes of the back and forward TE$_{10}$ scattered modes, in the manner of Stevenson [1], one can show that $B$ is related to the slot voltage $V_{n}^{S}$ by the equation

$$V_{n} = \frac{1}{Y_{n}^{A}/G_{0}} \left\{ j \left[ \frac{8}{\pi^{2} \eta G_{0}} \frac{(a/b)}{(\beta/k)} \right]^{1/2} \sin \frac{\pi x_{n}}{a}(\cos \beta l_{n} - \cos kl_{n}) \right\} V_{n}^{S}.$$  

It will be seen shortly that (7) is one of the two principal design equations which emerge from the analysis. A study of (7) reveals that the mode voltage and slot voltage are in phase quadrature if $Y_{n}^{A}/G_{0}$ is purely real. In most slot array design problems, $V_{n}^{S}$ is governed by the pattern requirements and $V_{n}$ is a common voltage in any given branch line. Thus, if all the $V_{n}^{S}$ slot voltages are to be in phase with each other, and all the mode voltages $V_{n}$ are to be in phase with each other, it follows that all the active admittances $Y_{n}^{A}$ should have a common phase. A simple choice is to require that all $Y_{n}^{A}$ be pure real. But a study of (4) indicates that, if $Y_{n}^{A}$ is to be pure real, in general $Y_{n}$, the self-admittance of the $n$th slot, will not be pure real. In other words, when mutual coupling is taken into account, one cannot expect the resonant self-admittance data to be pertinent in the design. Indeed, in many practical applications, the requisite value of $Y_{n}$ can be quite far off resonance.

The other principal design equation arises from linking the performance of the slot array to that of an equivalent dipole array via Babinet's principle. Clearly, if the usual assumption of an infinite perfectly conducting ground plane is made, and if the feeding currents of the center-fed strip dipoles match the slot voltages of the slots, the patterns will be essentially the same. To get the impedance characteristics to match also, one needs to place a load impedance $Z_{n}^{L}$ in series with the $n$th
defined by

\[
\frac{Y_n^A}{G_0} = \frac{1}{Z_n^A/73} \left( \frac{4(a/b)}{0.61\pi(\beta/k)} \cos \beta_n \right)
- \cos kl_n)^2 \sin^2 \frac{\pi x_n}{a} \right). \tag{8}
\]

In (8), \(Z_n^A\) is the active impedance of the \(n\)th strip dipole, defined by

\[Z_n^A = Z_n + Z_{nL} + \sum_{m=1}^{N'} \frac{I_m}{I_n} Z_{mn}, \tag{9}\]

wherein \(Z_n\) is the self-impedance of the dipole, \(Z_{nL}\) is the load impedance placed in series with it, \(Z_{mn}\) is the conventional mutual impedance between dipoles calculable from formulas such as those of Baker and Lagrone [3], and \(I_m/I_n\) is the aperture distribution. Thus if the pattern requirement is known (so that \(I_m/I_n\) is known), and if \((Z_n + Z_{nL})\) is known as a function of \(x_n\) and \(l_n\) (this relation will be deduced shortly), then \(Z_n^A\) can be calculated, placed in (8), and \(Y_n^A/G_0\) can be determined.

Equation (8) permits the interesting interpretation that the normalized active admittance of a longitudinal dipole is equal to Stevenson’s expression for the resonant normalized conductance (the factor in curly brackets) divided by the active impedance of the corresponding loaded dipole normalized to 73 Ω.

Equation (8) also applies for the case of an isolated slot, in which case \(Z_n^A\) reduces to \(Z_D + Z_{nL}\), with \(Z_D\) the self-impedance of the isolated strip dipole and \(Z_{nL}\) the load impedance in series with it whose presence models the reactive effects of internal higher order mode scattering off the slot due to its offset. This serves to point out some of the limitations of Stevenson’s original expression. Not only does it apply only for resonant length slots, but strictly it becomes a less accurate approximation as the slot width and/or its offset is increased. This is because \(Z_D\) is affected by the width of the strip dipole, and \(Z_{nL}\) is affected by the offset of the slot.

Equation (8) can be partitioned [2] to yield the first-order results

\[
\frac{Y_n}{G_0} = \left[ \frac{4(a/b)}{0.61\pi(\beta/k)} \cos \beta_n \right] \frac{(Z_{nn}/73) - \sum_{m=1}^{N'} (Z_{mn}/73)^2/((Z_{mm}/73))}{(Z_{nn}/73) - \sum_{m=1}^{N'} (Z_{mn}/73)^2/((Z_{mm}/73))}.
\]

\[
Y_{mn} = \frac{\cos \beta_m - \cos kl_m}{\cos \beta_m - \cos kl_m} \sin \frac{\pi x_n}{a}.
\]

Equation (11) leads to the interesting conclusion that

\[
Y_{mn} = \frac{Z_{mn}^2}{Z_{mn} Z_{mn}}.
\]

When use is made of (10)-(12), it is important to remember that \(Y_{mn}, Y_{mn}^2\) are admittances associated with the mode voltages in the slot array, and that, whereas \(Z_{mn}\) is the conventional mutual impedance between dipoles, \(Z_{mn}^2\) is the loaded self-impedance of the \(n\)th dipole since it contains \(Z_{nL}\).

**EXPERIMENT**

If the foregoing theory is valid, the proper design of a one- or two-dimensional longitudinal shunt slot array involves the choice of offsets and lengths for the various slots such that (7) and (8) are simultaneously satisfied for all values of \(n\). One begins by knowing the desired aperture distribution \(V_m/V_n\) for the slots, or \(I_m/I_n\) for the equivalent dipoles) and the relative mode voltages \(V_m/V_n\) (these would all be the same in a standing wave linear array, but would depend on the selection of main-line/branch-line coupling coefficients in a planar array). Then knowledge of the function \(X_{mn}^A(x_1, \ldots, x_n, l_1, \ldots, l_N)\) permits determination of all the lengths and offsets such that the desired aperture distribution is achieved, and such that the individual values of \(Y_{mn}/G_0\) cause the branch line admittances and main line admittances to add up to give the desired match.

A key ingredient in this process is to find the function \(Z_n^A(x_1, \ldots, x_N, l_1, \ldots, l_N)\). As mentioned earlier, the mutual part of \(Z_n^A\) can be calculated from conventional formulas if the aperture distribution is specified. Now we turn our attention to the determination of the self-part of \(Z_n^A\), namely \((Z_n + Z_{nL})\). If we assume that \((Z_n + Z_{nL})\) is essentially the same whether the other dipoles are present and open circuited, or absent, then \((Z_n + Z_{nL}) = (Z_{SELF} + Z_{LOAD})\); that is, it equals the loaded self-impedance of an isolated dipole (corresponding to an isolated slot). But for this case (8) becomes

\[
Z_{SELF} + Z_{LOAD} = \frac{73}{Y_{SELF}/G_0} \left[ \frac{4(a/b)}{0.61\pi(\beta/k)} \cos \beta_n \right]
- \cos kl_n)^2 \sin^2 \frac{\pi x_n}{a} \right]. \tag{13}
\]

Regardless of the shape of the slot (rectangular, rounded ends, dumbbell, etc.), if one measures \(Y_{SELF}/G_0\) as a function of offset \(x\) and length \(l\), (13) can be used to express \((Z_n + Z_{nL})\) as a function of \(x_n\) and \(l_n\). This can then be used in (8) for all aperture distributions and feeding arrangements. For rectangular slots, the theoretical values of \(Y_{SELF}/G_0\) obtained by the method of Khac [4] can be used in lieu of experimentally obtained information.

It is desirable to accumulate the data on \(Y_{SELF}/G_0\) in the universal form discovered by Stegen [5] and illustrated in Fig. 9-5 of Jasik [6]. This figure shows plots of the real and imaginary parts of \(Y_{SELF}/G_0 + G_{RES}/G_0\) versus \(l/R_{RES}\). The range of greatest use in the design of slot arrays is \(0.95 < l/R_{RES} < 1.05\) and the theoretical work of Khac [4] supports the assumption of universality in this range. Fig. 9-5 of Jasik requires his companion Figs. 9-6 and 9-7, in which \(G_{RES}/G_0\) and \(2l_{RES}/l_0\) are plotted as functions of slot offset. When polyfits are made
to the four curves in Fig. 9-5, 6, and 7 of Jasik, \((Z_n + Z_n^L)\) can be expressed in a form easily handled by a computer.

Fig. 9-7 of Jasik leads to a first test of the theory. Stegen dealt with round ended slots in a wall 0.050 in thick. The question arises as to the length of the equivalent strip dipole of rectangular contour in a wall of “zero” thickness. This can be determined by the following argument. As the offset \(x \to 0\), the amplitudes of all the modes scattered off the slot tend to zero. With respect to higher order mode scattering, this has the implication for the complementary dipole that its loading impedance tends to zero also. But in this circumstance, (13) indicates that \(Z_{SELF}^c\) should be pure real for the dipole when \(Y_{SELF}^c\) is pure real for the slot. Tai has shown \([7]\) that a strip dipole of width \(w\) and negligible thickness is equivalent to a cylindrical dipole of diameter \(d = w/2\). Tai also provides a convenient formula \([7]\) for the impedance of a cylindrical dipole as a function of its length \(2l\) and its radius \(a = d/2\). Since Stegen used slots 0.0625 in wide, if one places a 0.0156 in in Tai’s formula, one can deduce that \(2l/\lambda_0 = 0.464\), wherein \(2l\) is the resonant length of the unloading strip dipole. On the other hand, a study of Fig. 9-7 of Jasik reveals that Stegen’s asymptotic value is \(2l/\lambda_0 = 0.483\), in which \(2l\) is the resonant length of his round ended slot at zero offset. From this it follows that \(f = 2l/\lambda = 1.04\). This length adjustment factor is in agreement with the findings of Oliner \([8]\), who attributes a 2 percent correction for round ends and a 2 percent correction for wall thickness in this situation.

When the foregoing theory is used to design slot arrays, the procedure just described can be utilized to determine the length adjustment factor \(f\). Experience shows that \(f\) is quite sensitive to the \(b\) dimension of the waveguide, as well as to wall thickness.

A second test involves a prediction of resonant conductance versus offset for an isolated slot. Since the higher order mode scattering off this slot is nonpropagating and thus contributes primarily to the storage of reactive energy, it seems reasonable to assume that the load impedance \(Z_L\) possesses a small resistive component \(R_L\). In practical circumstances, the dipole self-impedance \(Z_D\) has a resistive component in the neighborhood of 73 \(\Omega\), and thus one should expect that \(R_L \ll R_D\). For a resonant slot \(X_L = -X_D\), and in this case (8) can be approximated by

\[
\frac{G_t}{G_0} = \frac{73}{R_D} \left( \frac{4(a/b)}{0.61\pi(\beta/k)} \right) \left( \frac{\cos \beta l_r - \cos k \lambda}{\sin^2 \frac{\pi x}{a}} \right) f. \quad (14)
\]

For standard X-band guide, a frequency of 9.375 GHz, and a length adjustment factor \(f = 1.04\), (14) yields the solid curve found in Fig. 3. Stegen’s experimental points are shown for comparison.

It should be recognized that the agreement seen between theory and experiment in Fig. 3 is not a case of adjusting a parameter in the theoretical formula to get curve fitting. All that has been done in (14) is to ignore \(R_L\) and assume that the equivalent dipole is resonant. A plot of the original Stevenson formula would lie 20 percent below the solid curve of Fig. 3 at the low end, and 10 percent below it at the high end.

A third test involving an isolated slot concerns the frequency dependence of resonant conductance. Stegen \([5]\) found experimentally that his curve of resonant length versus offset for a longitudinal shunt slot (Fig. 9-7 of Jasik) is universal in the sense that if the offset remains constant, \(2l/\lambda_0\) also remains essentially constant even though the frequency varies. This has the implication that if \(k\lambda\) remains constant in (14), that is, the slot length is continually adjusted as the frequency is changed so as to maintain resonance, then for a slot of a given offset, \(G_t/G_0\) is a function of frequency only because \(\beta/k\) varies with frequency. For a slot of offset 0.183 in, (14) yields the solid curve shown in Fig. 4. Stegen’s experimental data points are shown for comparison.

Now let us consider situations involving more than just one isolated slot. As a first step, an array of two slots, one each in two parallel waveguides, with the slots staggered longitudinally a quarter of a guide wavelength, was constructed with the dimensions shown in Fig. 5 and imbedded in an 8-in by 10-in ground plane. This array was used to test the validity of (10)-(12) in the following way. With one slot covered over
with conducting tape, a short circuit was placed $3\lambda_g/4$ beyond the other slot, and a measurement was taken of its input admittance. This resulted in the data shown in the first two rows of Table I. A length contraction factor $f = 1.03$ was found to apply for this configuration and used to determine $l_1$ and $l_2$. Equation (8) then gave

$$Z_{11} = \frac{33.76}{Y_1 / G_0 \text{ (isolated)}}$$

$$Z_{22} = \frac{43.72}{Y_2 / G_0 \text{ (isolated)}}$$

from which the entries in the third and fourth rows of Table I were obtained.

$Z_{11}$ and $Z_{22}$ as they appear in (15) are the loaded self-impedances of the strip dipoles equivalent to each isolated slot. Strictly speaking, they are not the same as the quantities one should use when other dipoles are present but open-circuited; however, at this slot spacing the approximation is a good one, and therefore the entries for $Z_{11}$ and $Z_{22}$ in Table I will be used in (10)–(12).

The calculation of mutual dipole impedance was made using the formulas of Baker and LaGrone [3], and provides the fifth row entry in Table I.

Equation (10) predicts that if the conducting tape covering the second slot is removed and replaced by a short circuit $\lambda_g$ from the center of the second slot, and then the input admittance of the first slot is measured, the result should satisfy

$$\frac{Y_1}{G_0} = \frac{33.76}{Z_{11} - (Z_{12}^2/Z_{22})}$$

$$\frac{Y_2}{G_0} = \frac{43.72}{Z_{22} - (Z_{12}^2/Z_{11})}.$$
uniform phase, so the predicted pattern has a broadside beam, symmetrical sidelobes, and a 13.5-dB sidelobe level. The experimental H-plane pattern is shown in Fig. 8.

The range of lengths and offsets found for this two-by-four array illustrates the general observation that small arrays present a more difficult design problem than do large arrays. In the latter, only elements near an edge “see” a different mutual coupling environment, so achieving the proper active admittance becomes simpler. Further, mechanical and electrical tolerances ease off as the array gets larger [9].

Though the details are not being reported here, the above procedure has been used successfully to design a 12-slot linear array for a 30 dB side lobe level, a 19-slot linear array for asymmetric side lobes (all at 20 dB except the inner three on one side of the main beam at 30 dB) and a 52-element two-dimensional slot array with a uniform aperture distribution.

**CONCLUSIONS**

A theory has been presented which can account for the array behavior of longitudinal shunt slots in terms of the characteristics of complementary dipoles. Formulas for active, self-, and mutual admittances of longitudinal slots have been derived. Slot arrays can be designed by choosing the lengths and offsets of individual slots such that (7) yields a slot voltage distribution consistent with the desired pattern, and such that (8) yields an active admittance distribution consistent with the feed and match requirements of the array.

The analysis can be repeated, in a step-by-step analog, for the case of inclined series slots in the broad wall of rectangular waveguides. It can also be extended to arrays of strip-line-fed slots.

The theory has been tested experimentally in a variety of situations involving a single slot, a pair of slots, and a small two-dimensional array. In general, the agreement has been found to be quite satisfactory.

**REFERENCES**


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