Mysteries of the Smith Chart

Transmission Lines, Impedance Matching, and Little Known Facts

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Outline

- ***Transmission Line Theory***
  - ➤ Historical development
  - ➤ Heaviside's rewrite of Maxwell's theory, Telegrapher's equations,
  - ➤ Impedance, reflection coefficient, SWR, phase constant, & velocity factor
  - ➤ Special facts for $\lambda/2$, $\lambda/4$, and $\lambda/8$ lossless lines
- ***The Smith Chart***
  - ➤ Bilinear complex functions
  - ➤ Impedance and admittance coordinates (circles, circles, and more circles)
- ***Impedance Matching***
  - ➤ Why match? Impedance matching vs. conjugate impedance matching
  - ➤ Single frequency matching
  - ➤ Multiple-frequency and broadband matching

Part 1: Transmission Line Theory

Key Dates in Electrical Transmission

1830's magnetic telegraphs - Gauss, Henry

1839 electromagnetic telegraph - Wheatstone & Cook

1844 telegraph in America - Morse

1850's thousands of miles of telegraph line U.S. and Europe

1851 40 mile cable under English Channel

1855 distributed analysis of transmission line - Lord Kelvin

1858 transatlantic cable, project delayed by civil war

1873 theory of electrodynamics - Maxwell

1876 invention of telephone - Bell

1880's vectors, vector calculus, reformulation of Maxwell's theory, transmission line theory - Heaviside

1886 experimental confirmation of Maxwell's Theory - Hertz

1937 early Smith Chart, published 1939 & 1944 - Smith

Heaviside's Vector Formulation of Maxwell's Theory

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial \mathbf{t}}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

"And God said, Let there be light; and there was light." Genesis 1:3

Frequency Domain or Phasor Form

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

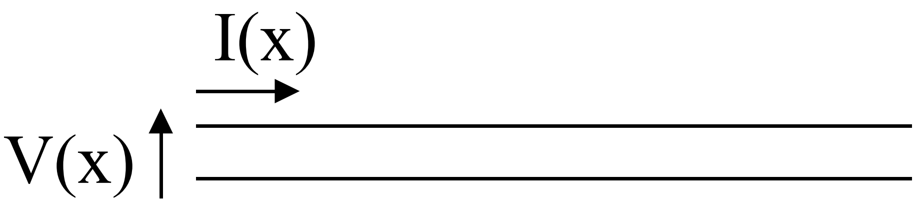
$$\nabla \times \mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E}$$

$$\nabla \cdot \mathbf{E} = \mathbf{0}$$

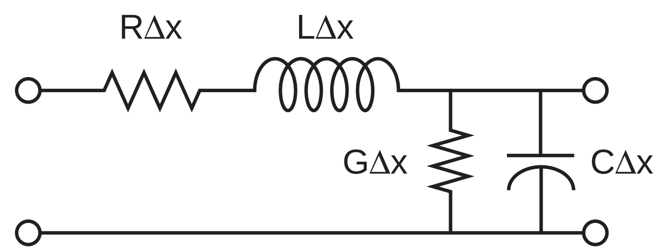
$$\nabla \cdot \mathbf{H} = \mathbf{0}$$

Heaviside's Telegrapher's Equations

Uniform transmission line



Equivalent circuit of infinitesimal segment



$$\frac{dV}{dx} = -(R + j\omega L)I(x)$$

$$\frac{dI}{dx} = -(G + j\omega C)V(x)$$

Transmission Line Solution
TEM Waves

Traveling wave

$$V(x) = \mathrm{V_o} e^{\gamma x}$$

$$I(x) = \frac{V(x)}{Z_o}$$

Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Characteristic impedance

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Notations
Real Parameters

R = series resistance per unit length (Ohms/meter)

L = series inductance per unit length (Henries/meter)

G = shunt conductance per unit length (Siemens/meter)

C = shunt capacitance per unit length (Farads/meter)

α ***= attenuation constant (nepers/meter)***

β ***= phase constant (radians/meter)***

λ ***= wavelength (meters)***

v_f ***= velocity factor (dimensionless)***

X = reactance (Ohms)

B = susceptance (Siemens)

s = standing wave radio (dimensionless)

Notations (Cont'd)
Complex Parameters

$Z = R + jX$ = impedance (Ohms)

Z_L = load impedance (Ohms)

Z_i = input impedance (Ohms)

Z_0 = characteristic impedance (Ohms)

$z = Z/Z_0 = r + jx$ = normalized impedance (dimensionless)

$Y = G + jB$ = admittance (Siemens)

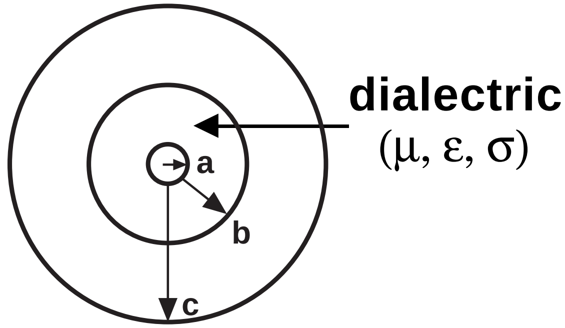
$y = Y/Y_0 = g + jb$ = normalized admittance (dimensionless)

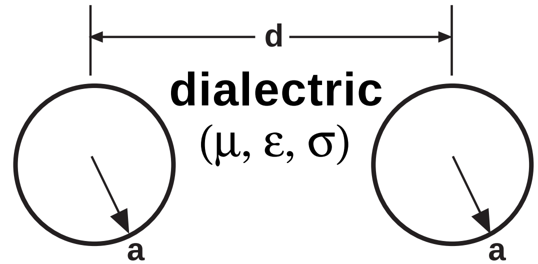
$\Gamma = \Gamma_r + j\,\Gamma_i$ = complex reflection coefficient (dimensionless)

$\gamma = \alpha + j\beta$ = propagation constant (inverse meters)

Transmission Line Parameters

Physical Dimensions and Material Properties





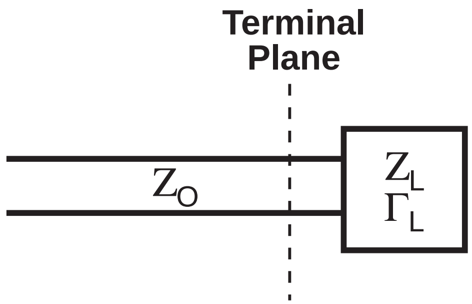
Parameter	*Coax*	*Twinlead*
R Ω/m	$\frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{1}{\pi a\delta\sigma_c}$
L H/m	$\frac{\mu}{2\pi}\left[\ln\frac{b}{a}+\frac{\delta}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right]$	$\frac{\mu}{\pi}\left[\frac{\delta}{2a}+\cosh^{-1}\frac{d}{2a}\right]$
G S/m	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$
C F/m	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$

where skin depth is $\delta = \frac{1}{\sqrt{\pi f\mu\sigma_c}}$

for copper $\sigma_c = 5.8\times10^{7}\ \text{S/m}$

$$\delta = \begin{cases} 8.5\ \text{mm at 60 Hz} \\ 6.6\ \mu\text{m at 100 MHz} \end{cases}$$

Reflection Coefficient and Impedance Relation at a Terminal Plane



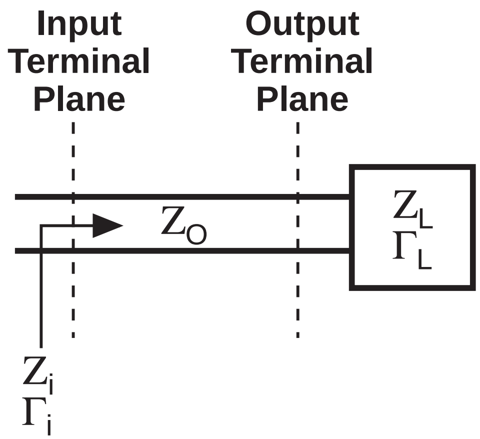
Definition

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{z-1}{z+1}$$

Inverse

$$z = \frac{1+\Gamma}{1-\Gamma}$$

- ***For every terminal plane, the complex load impedance and complex reflection coefficient seen to the right give the same information for that terminal plane***
- ***Question: How do Γ and z change as the terminal plane moves?***



Impedance relation

$$z_i = \frac{z_L + j \tan \beta l}{1 + j z_L \tan \beta l}$$

Cross relations

$$z_i = \frac{1 + \Gamma_L e^{-j2\beta l}}{1 - \Gamma_L e^{-j2\beta l}}$$

$$z_L = \frac{1 + \Gamma_i e^{j2\beta l}}{1 - \Gamma_i e^{j2\beta l}}$$

Reflection coefficient relation

$$\Gamma_i = \Gamma_L \, e^{-j2\beta l}$$

Wavelength

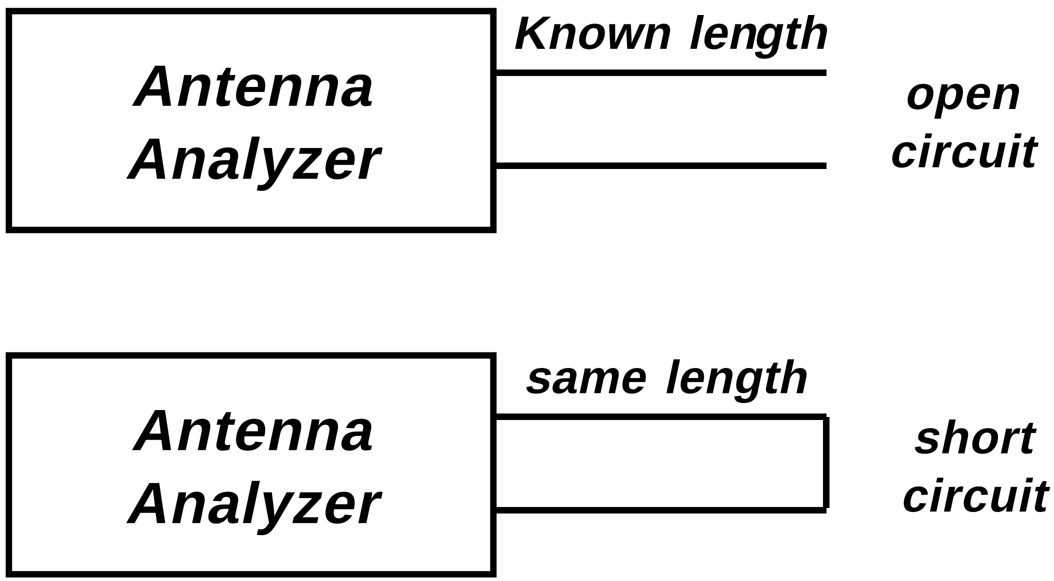
$$\lambda_{free\ space} = \frac{c}{f}$$

$$\lambda_{actual} = \frac{v}{f}$$

Velocity factor

$$v_f = \frac{v}{c} = \frac{\lambda_{actual}}{\lambda_{free\ space}}$$

How To Measure Velocity Factor of a Line
(One Way To Do It)



$$v_f = \frac{2\pi f l}{c} \frac{1}{\cot^{-1}\sqrt{\dfrac{-Z_{open}}{Z_{short}}}}$$

Phase Constant

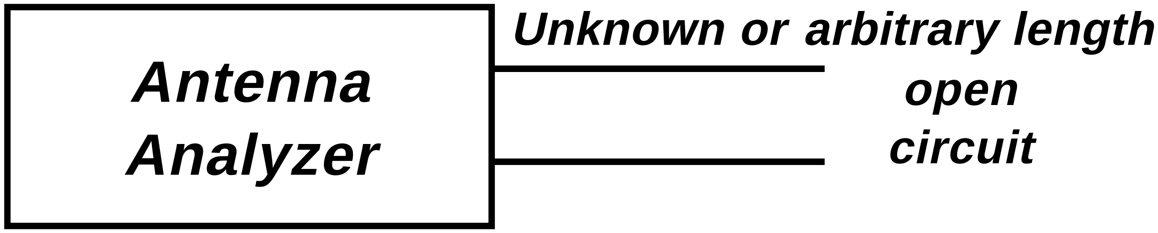
$$\beta = \frac{2\pi}{\lambda_{actual}} = \frac{2\pi f}{v_f \ c} \quad \text{radians/meter}$$

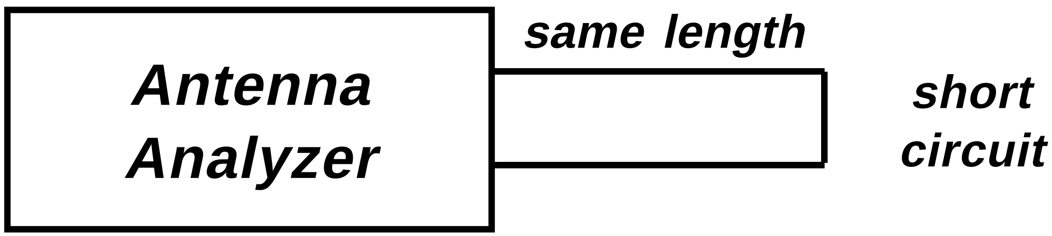
- ***Phase constant β and velocity factor v_f give equivalent information***
- ***Both can be calculated from line dimensions and material properties***

$$\beta = \mathrm{Im}\sqrt{(R + j\omega L)(G + j\omega C)}$$

- ***Best to measure!***

How to Measure Complex Z_o of A Line (One Way to Do It)





$$Z_o = \sqrt{Z_{open} \times Z_{short}}$$

- ❑ ***Geometric mean of two complex numbers***
- ❑ ***Calculation is trivial in polar form on Smith Chart***

What Special Lengths of Lossless Line Do

Half wavelength, $l = \lambda/2$

$$Z_i = Z_L$$

Quarter wavelength, $l = \lambda/4$

$$Z_i = \frac{Z_o^2}{Z_L}$$

Eighth wavelength, $l = \lambda/8$

$$|Z_i| = Z_o \qquad \text{if } Z_L \text{ and } Z_o \text{ are real (resistive)}$$

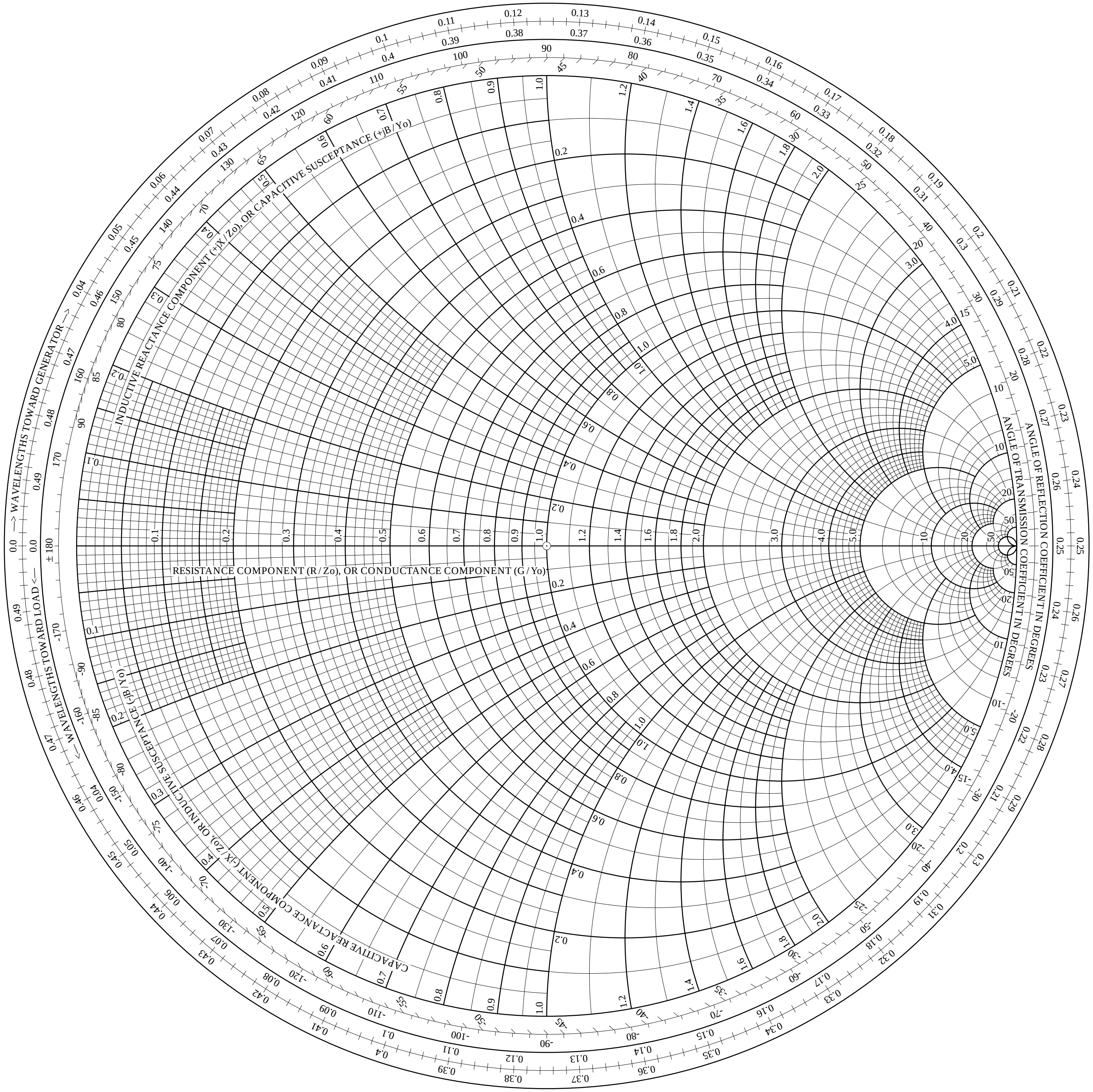
$$\boxed{\begin{aligned} s &= \frac{1+|\Gamma|}{1-|\Gamma|} \\[2em] |\Gamma| &= \frac{s-1}{s+1} \end{aligned}}$$

Easy to remember from

$$z = \frac{1+\Gamma}{1-\Gamma}$$

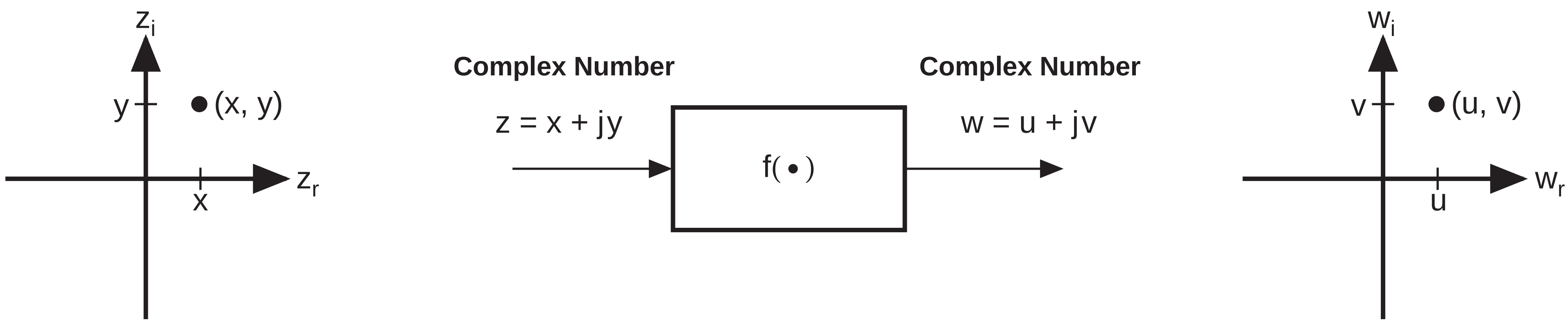
$$\Gamma = \frac{z-1}{z+1}$$

Part 2: The Smith Chart

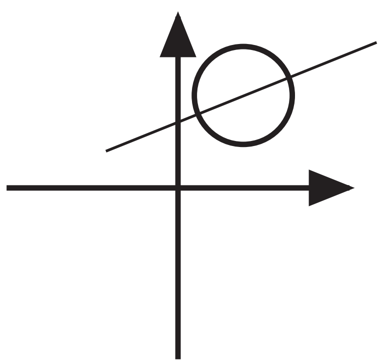


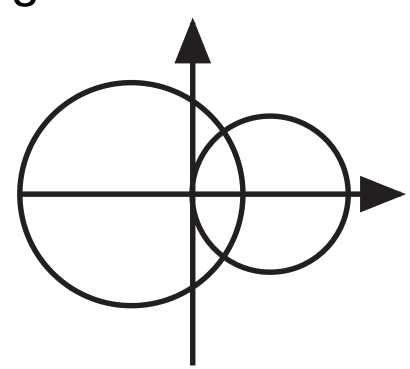
Pacificon '99

Complex Functions



- ***Basic types of complex functions***
  - Global Properties
    - Linear – lines map to lines
    - Bilinear – circles map to circles
  - Local Properties
    - Conformal – right angles map to right angles



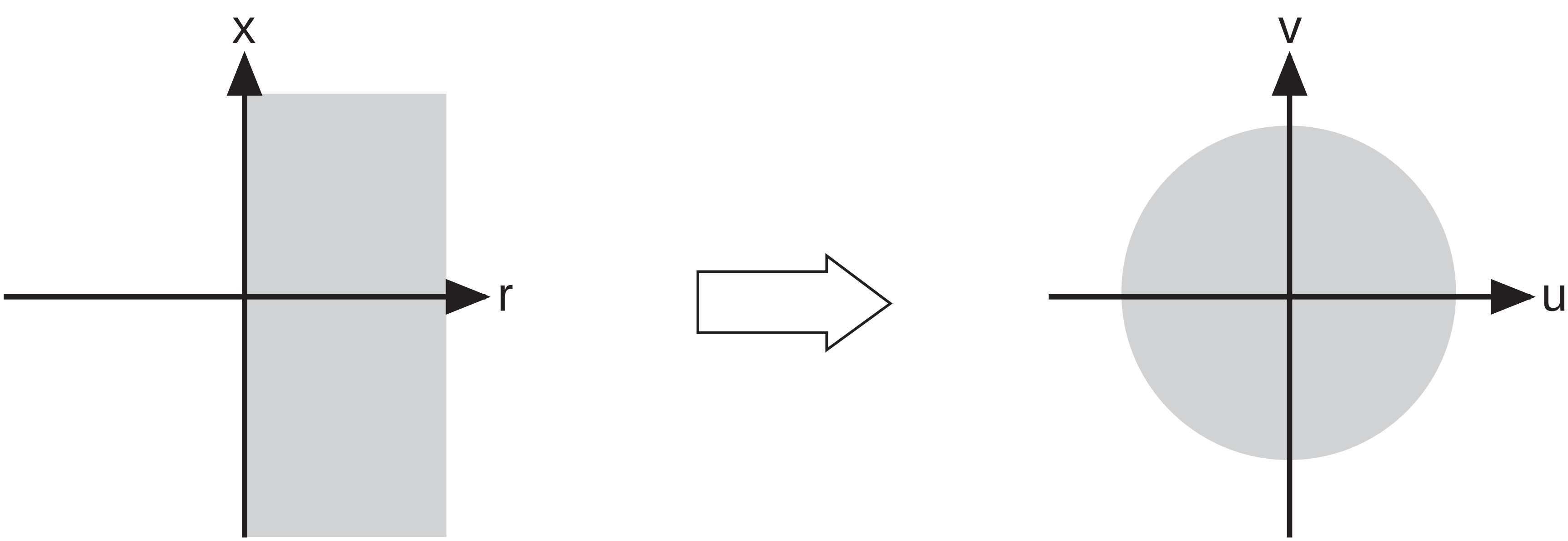


Mathematical Basis of the Smith Chart

$$\Gamma = \frac{z-1}{z+1}$$

a bilinear conformal complex function

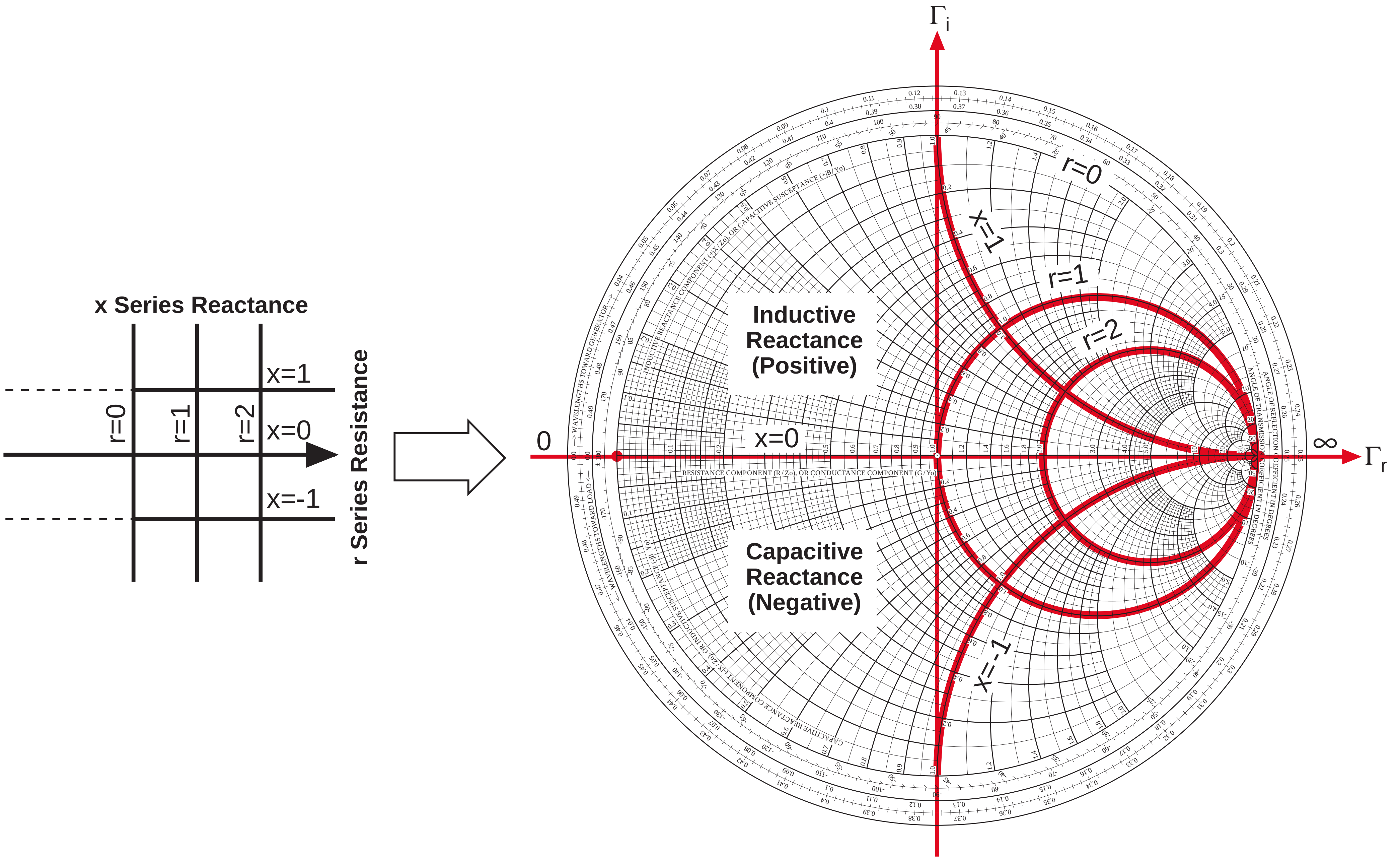
$$u + jv = \frac{(r-1) + jx}{(r+1) + jx}$$



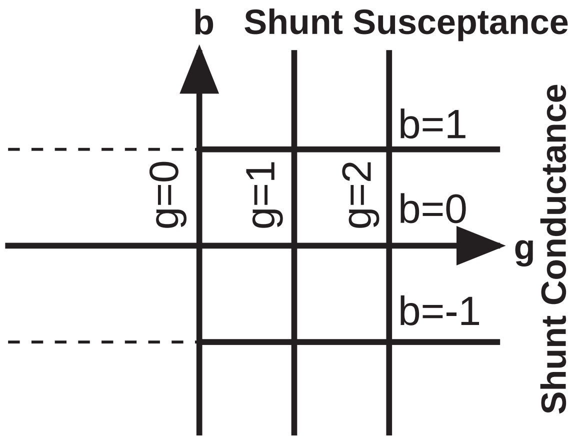
Right Half z Plane

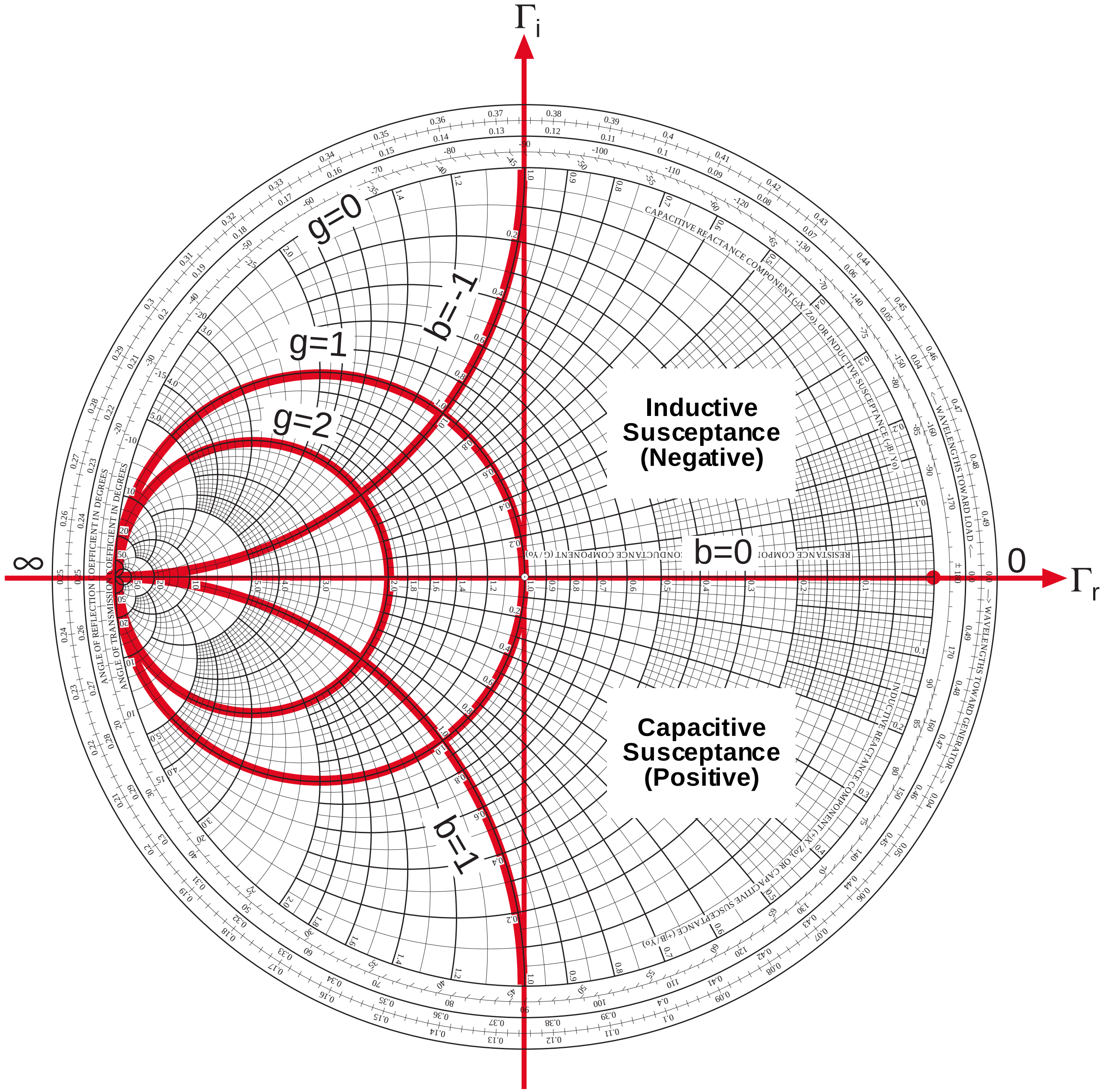
Interior Unit Circle Γ Plane

Smith Chart: Impedance Coordinates

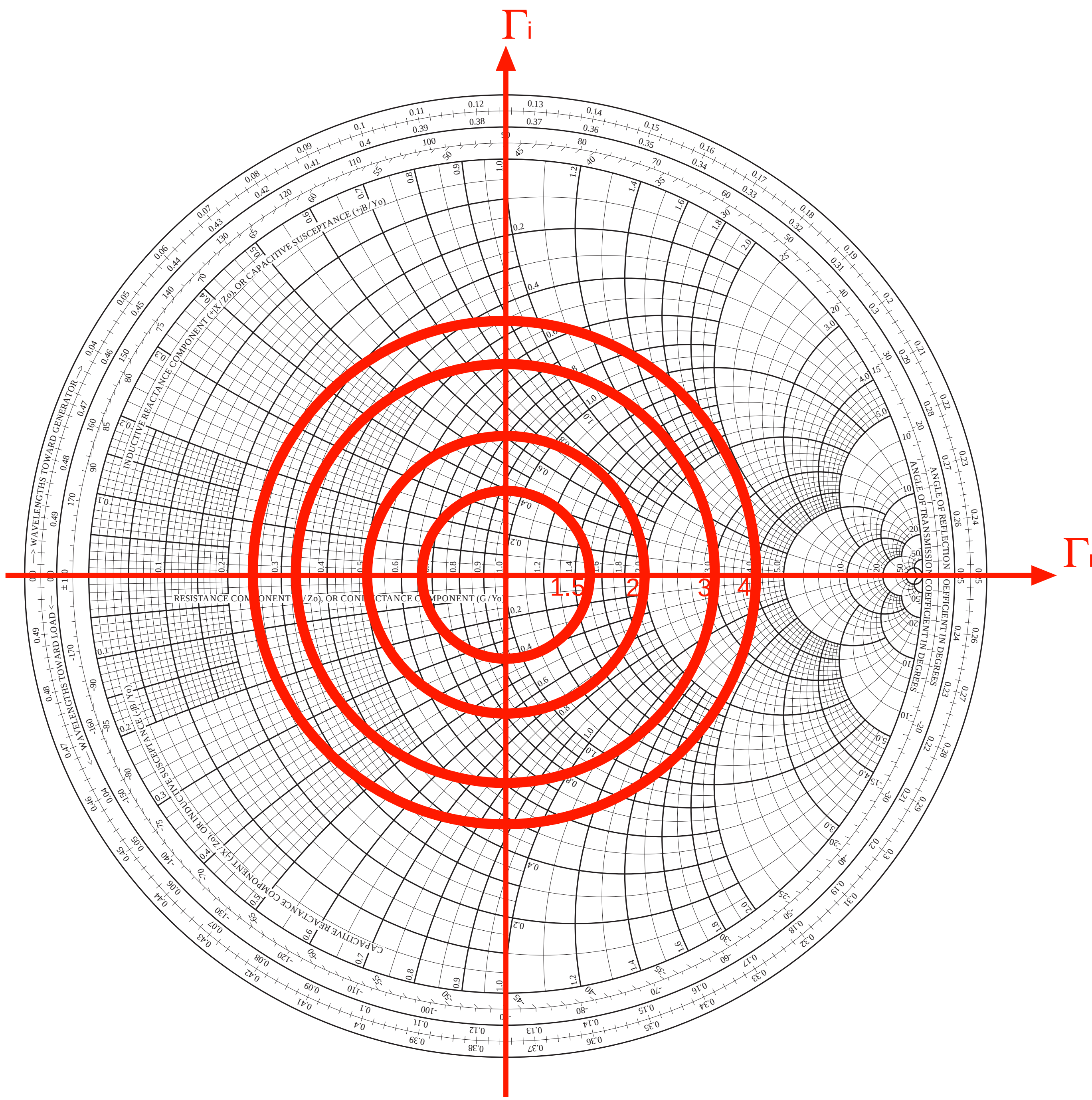


Smith Chart: Admittance Coordinates

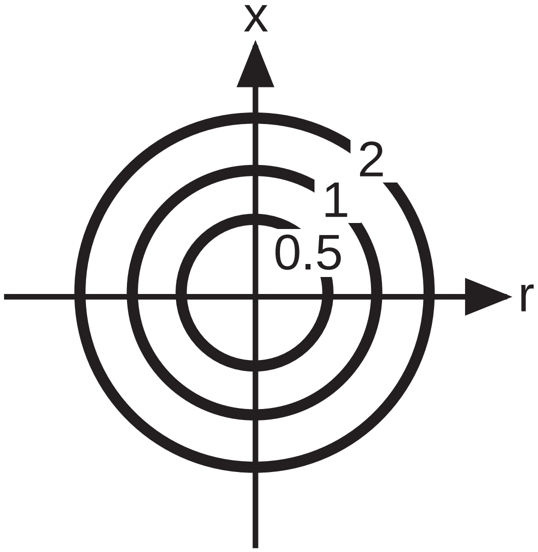


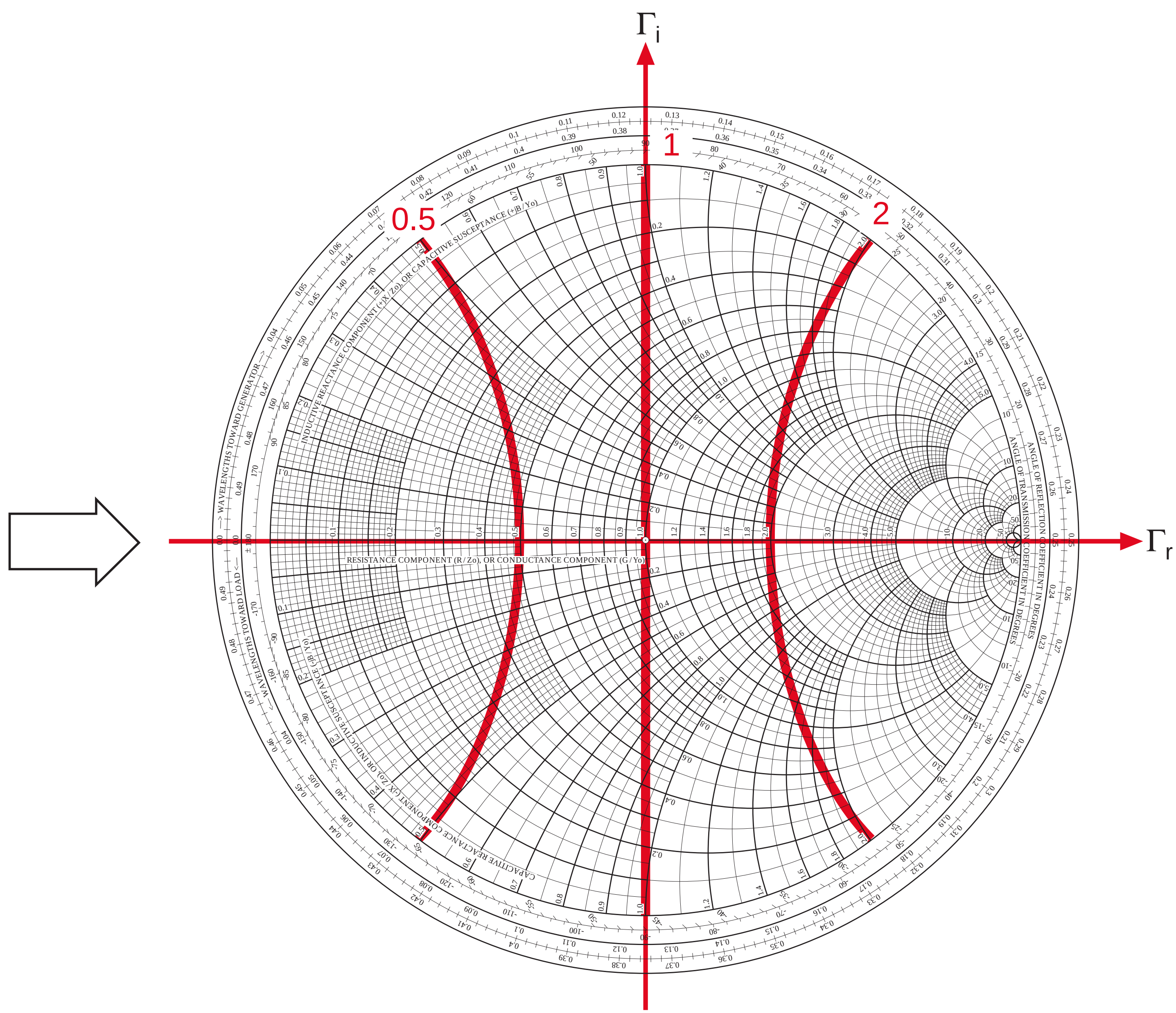


Smith Chart: Constant SWR Circles

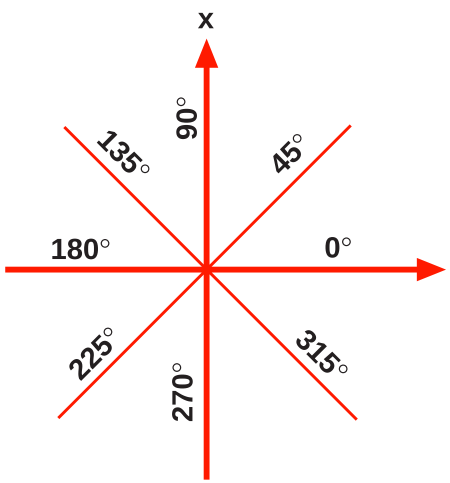


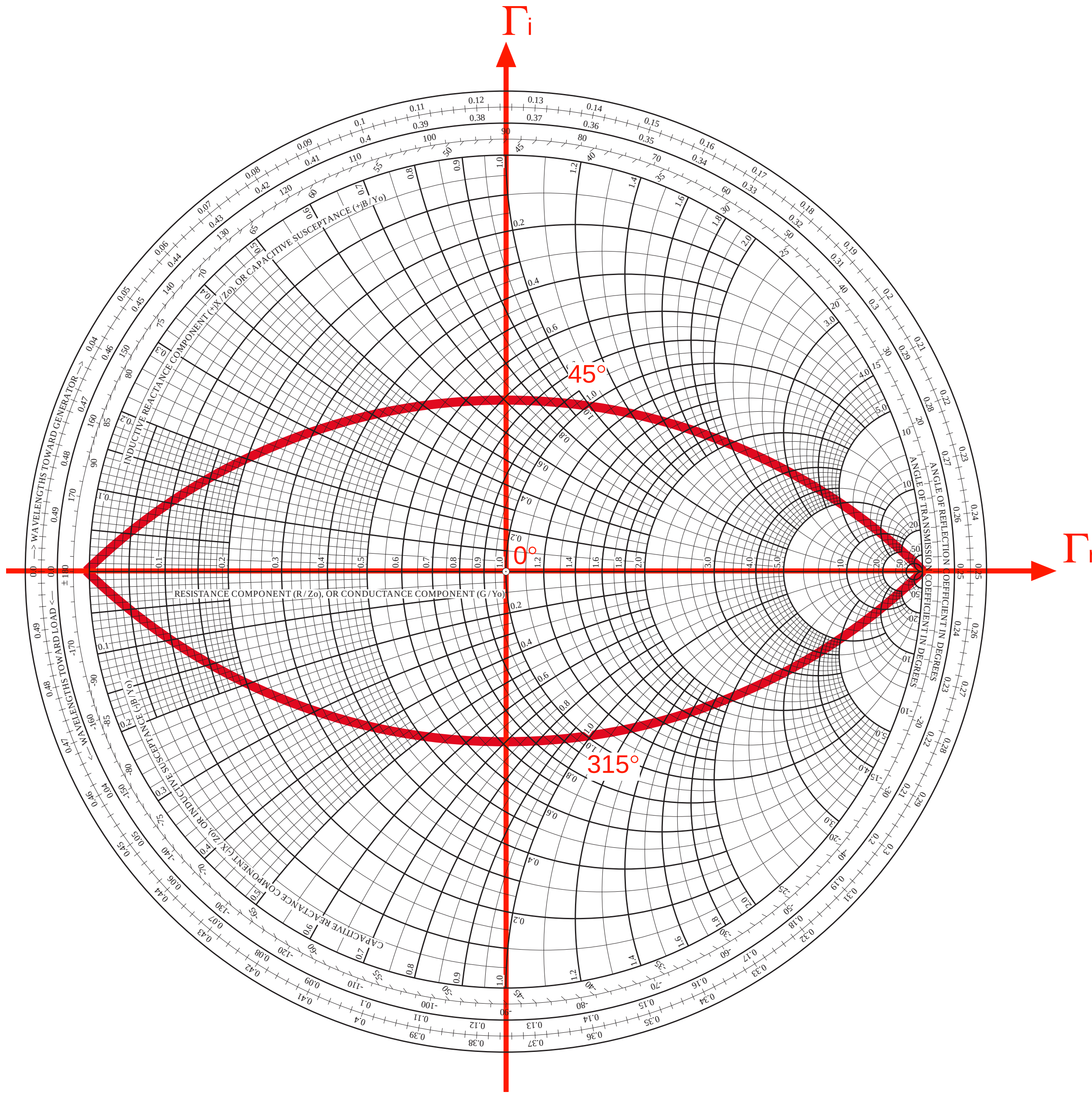
Smith Chart: Constant Impedance Magnitude Circles





Smith Chart: Constant Impedance Phase Angle Circles





Smith Chart: Multiplication, Division, Squares, & Square Roots

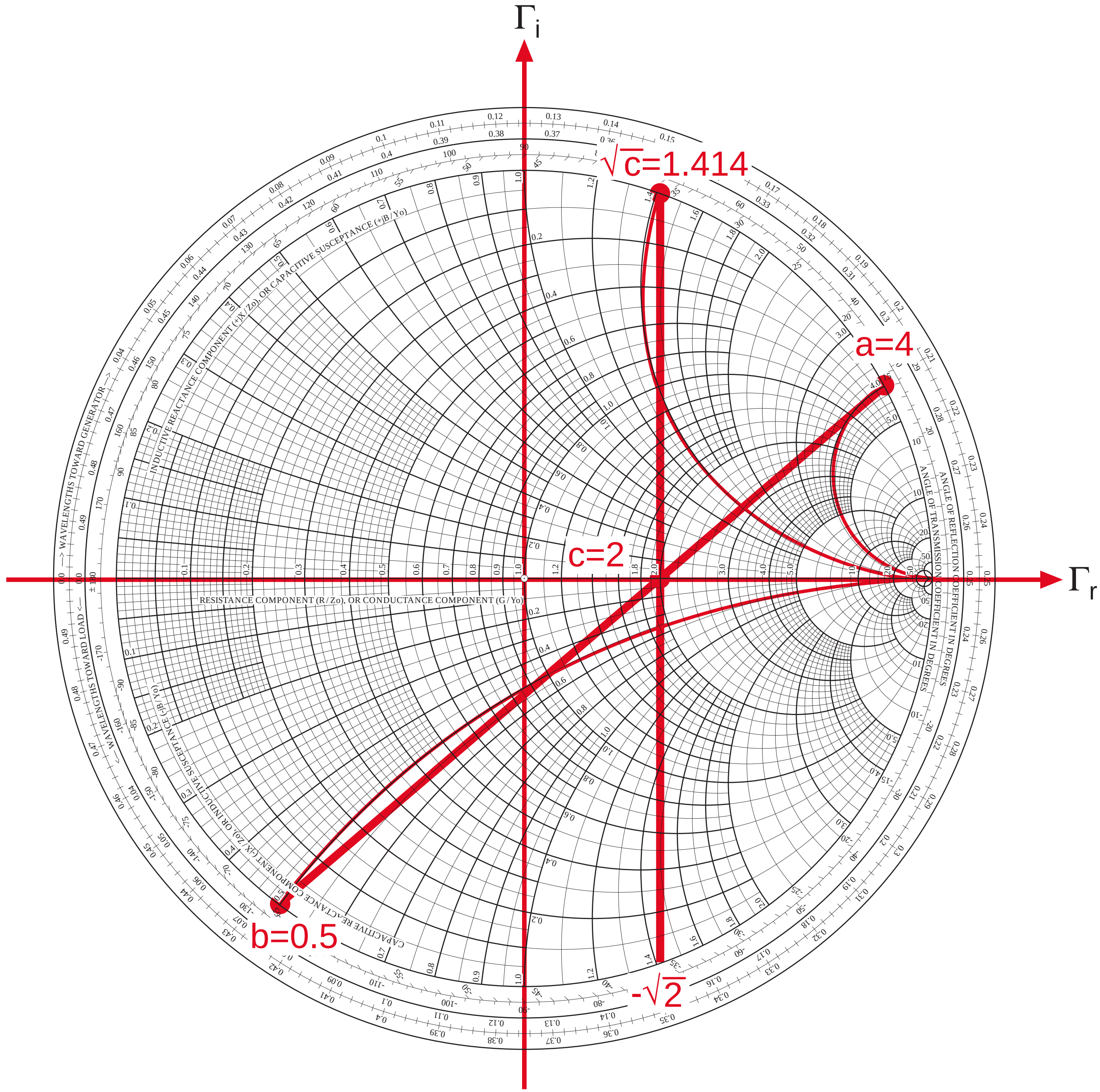
Unary Operators

squares	a^2
square roots	$\sqrt{a}$
tangents	$\tan\theta$
cotangents	$\cot\theta$
inverse tangents	$\tan^{-1} a$
Inverse cotangents	$\cot^{-1} a$

Binary Operators

multiplication X	$a \bullet b$
division ÷	c/a
geometric mean	$\sqrt{ab}$

Smith Chart: A Nomogram for Math Calculations



Part 3: Impedance Matching

Impedance Matching Categories

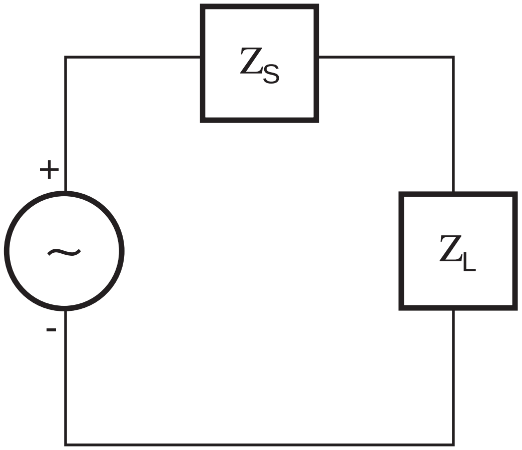
- ❑ ***Single frequency matching***
  - ➤ Manual synthesis using Smith Chart
  - ➤ Eight canonical networks
  - ➤ Lumped elements
  - ➤ Series and parallel stubs
  - ➤ Transmission line sections
- ❑ ***Multiple frequency matching***
  - ➤ Ladder networks
  - ➤ Multiple stubs
  - ➤ Multiple line sections
- ❑ ***Broadband matching***
  - ➤ Maximize SWR bandwidth
  - ➤ Numerical optimization
  - ➤ winSMITH software
  - ➤ Smith Chart used for visualization only

Bandwidth Classifications

	Fractional Bandwidth
Narrowband	***< 10%***
Moderate band	***10% to 50%***
Broadband	***> 50%***

Two Kinds of Matching

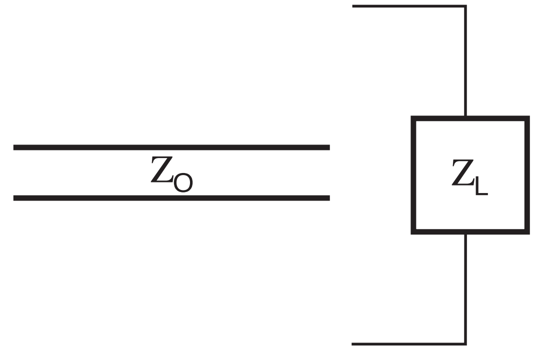
- ***Conjugate Matching***



$$Z_s = Z_L{}^*$$

- ***Best use at source (transmitter)***
- ***Maximizes power delivery to the load***
- ***Does not minimize reflections unless Z_s is real***
- ***Normally done by the transmitter manufacturer at the circuit design level***
- ***Ideally Z_s (ext) = 50 Ω***

- ***Load Matching***



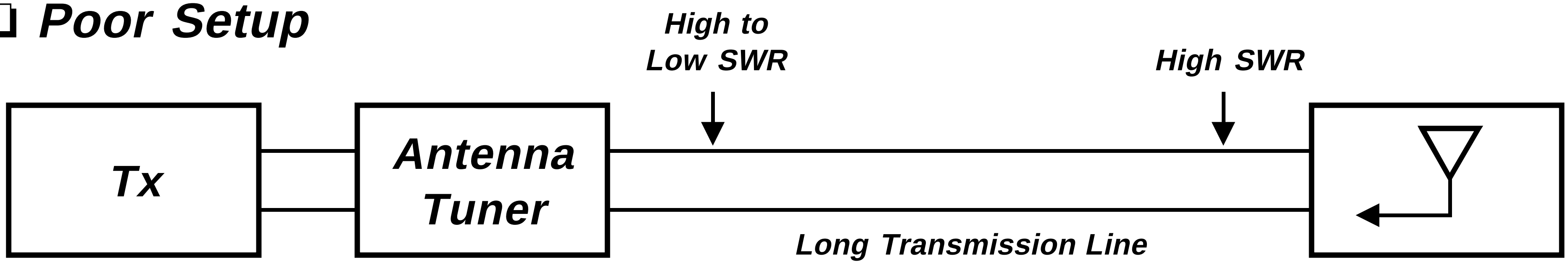
$$Z_L = Z_O$$

- ***Best used at ends of transmission lines***
- ***Minimizes reflections***
- ***Does not maximize delivered power unless Z_0 is real***

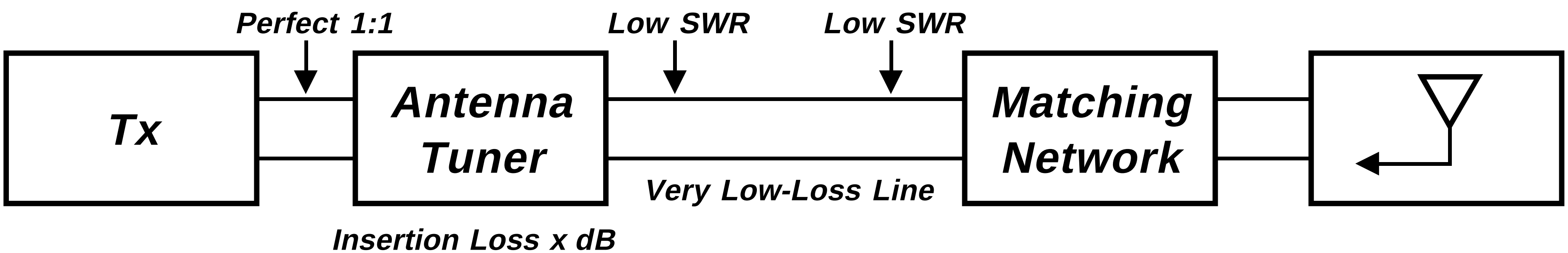
Where Should Matching Network Go?

TRW

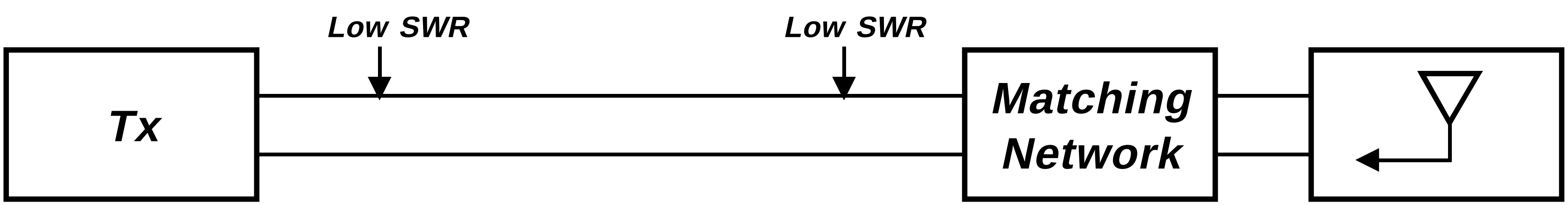
❑ Poor Setup



❑ Good Setup



❑ Best Setup



Necessary Test Equipment

- ***Antenna analyzer***
  - *Autek*
  - *CIA*
  - *MFJ*
- ***Noise bridge (less accurate)***
- ***Network analyzer (more accurate)***
  - *Hewlett-Packard*

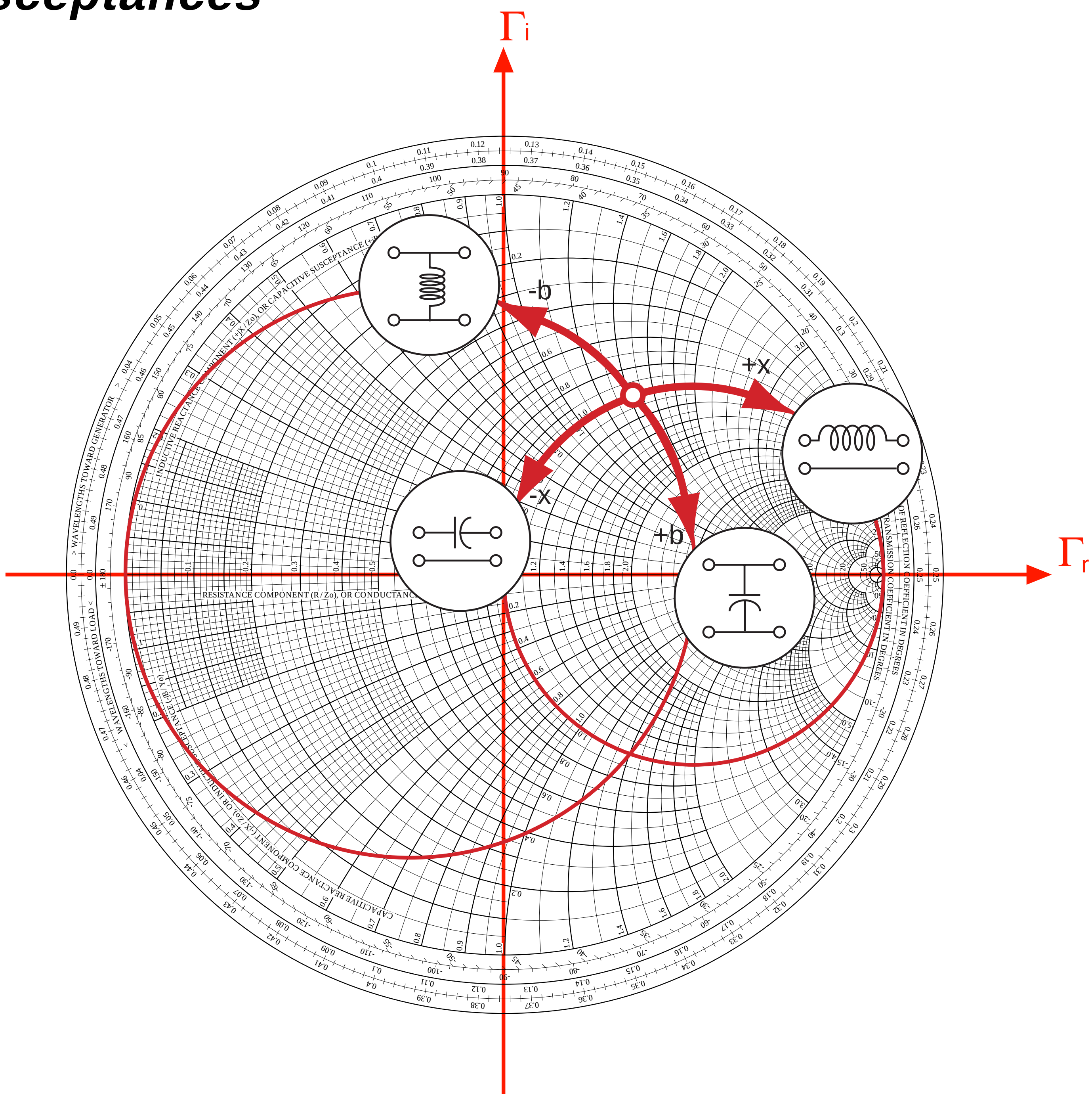
Matching Network Design Recipe

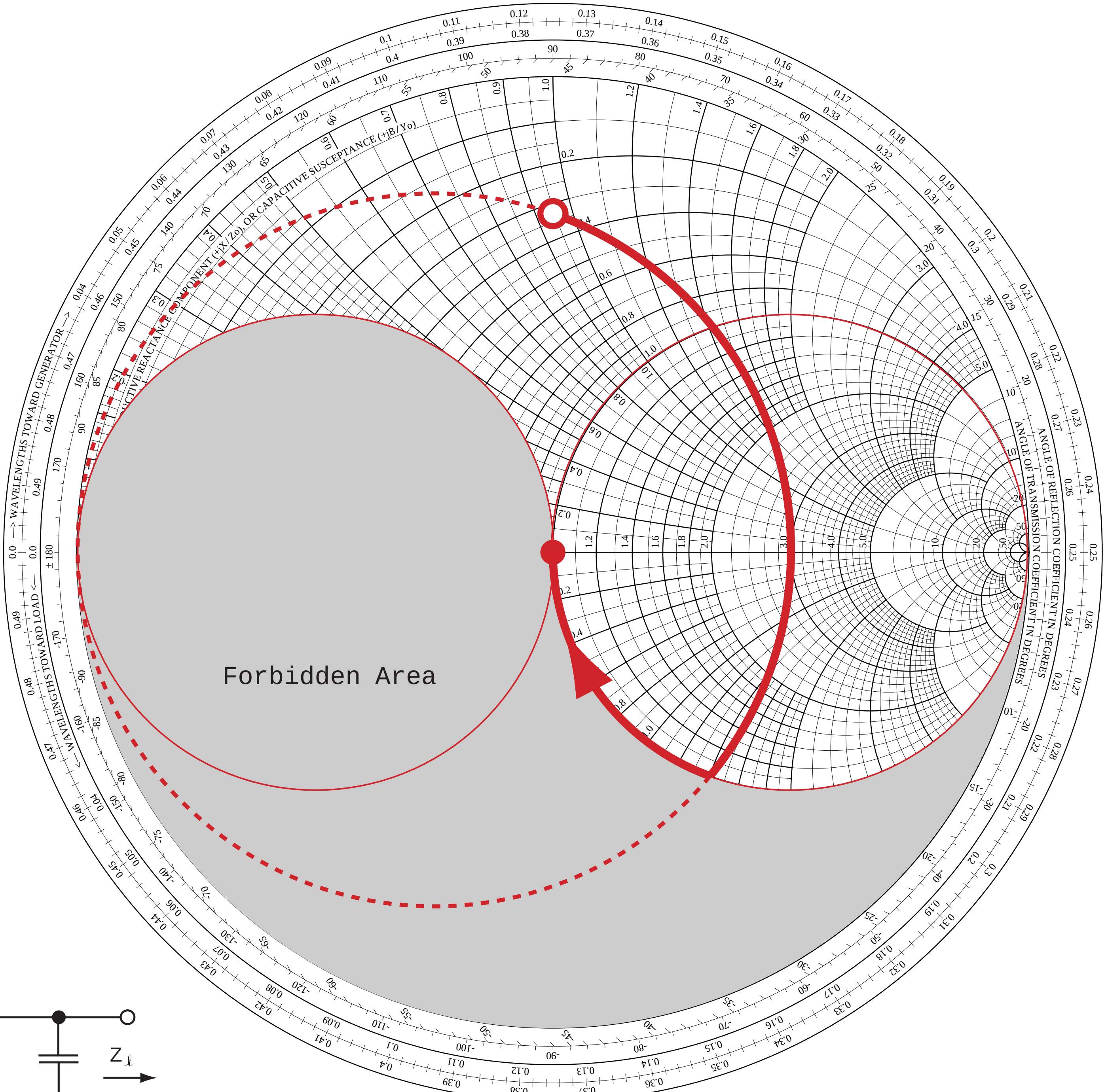
1. ***Measure transmission line parameters Z_o, v_f***
2. ***Measure antenna feedpoint impedance across band(s) of interest***
3. ***Measure or calculate impedance across band(s) of interest at network insertion point***
4. ***Narrowband match:***
   - ➤ *Select appropriate lossless L network, 2 or 4 choices*
   - ➤ *Select lumped elements vs stubs*
   - ➤ *Calculate component values*
   - ➤ *Calculate SWR and SWR bandwidth*
   - ➤ *Build and test*
5. ***Broadband match:***
   - ➤ *Use design software - winSMITH or equivalent*
   - ➤ *Design n-stage lossless ladder network*
   - ➤ *Select lumped elements vs stubs*
   - ➤ *Calculate component values*
   - ➤ *Calculate SWR and SWR bandwidth*

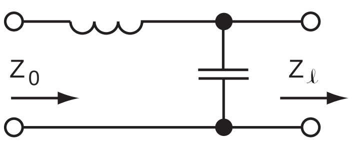
How to Measure Antenna Feedpoint Impedance

- ***Measure impedance through known line***
- ***Divide measure impedance by Z_o***
- ***Plot impedance point on Smith Chart***
- ***Move counter clockwise on chart by electrical length of line***
- ***Read coordinate values from chart***
- ***Multiply result by Z_o***

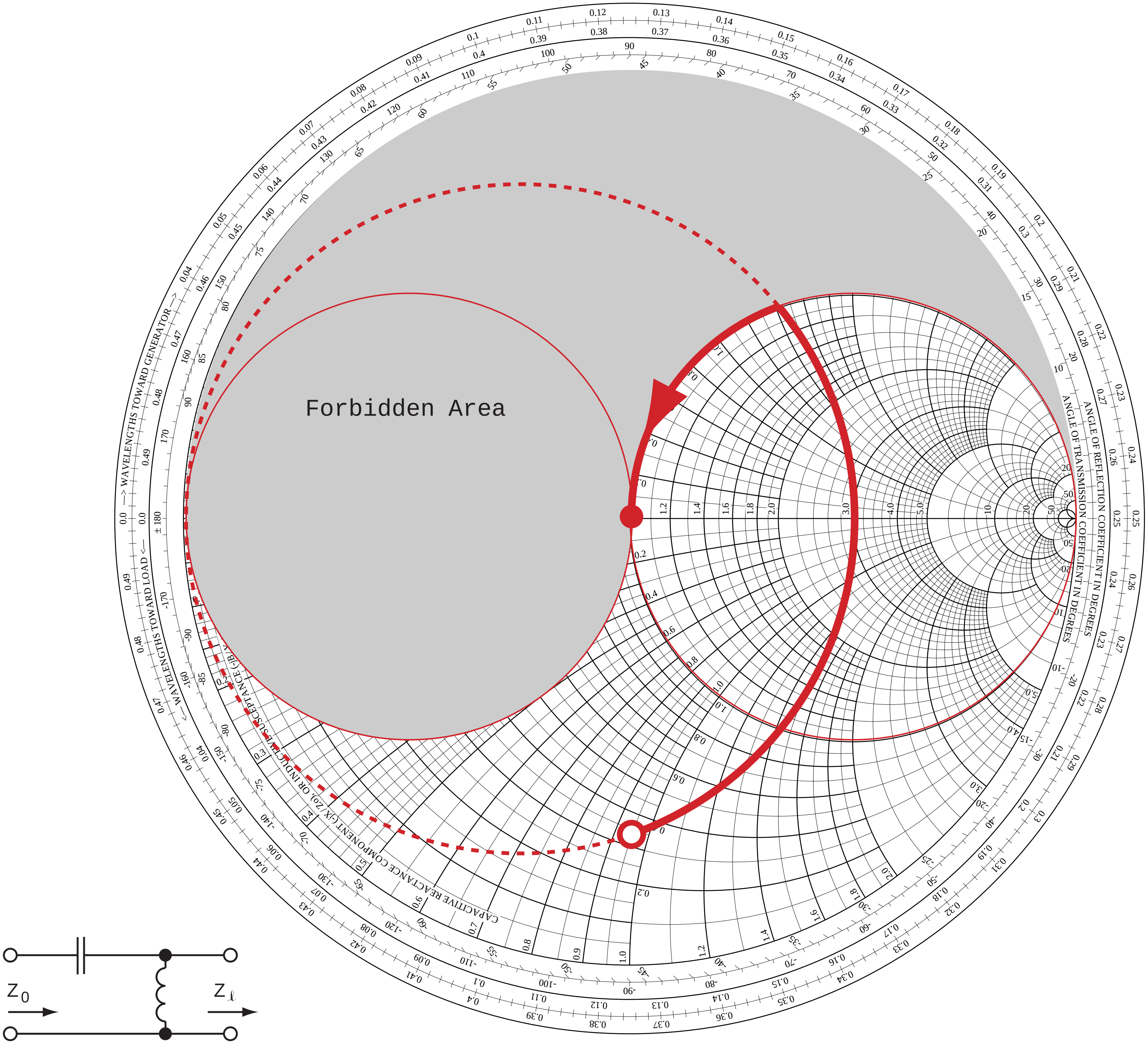
Smith Chart: Effect of Adding Series Reactances or Shunt Susceptances





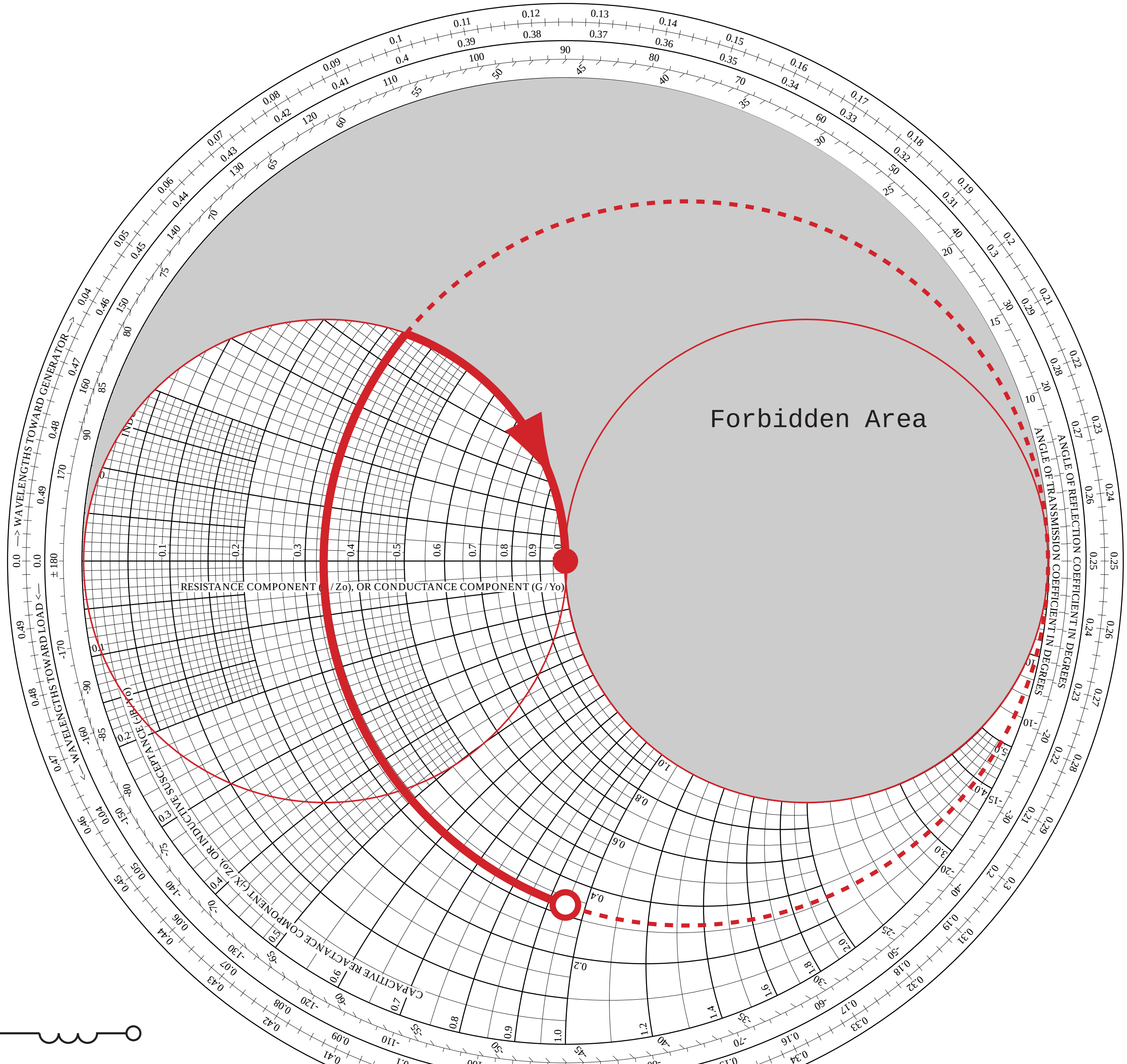


(a)

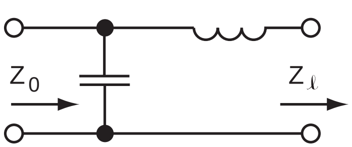


(b)

TRW

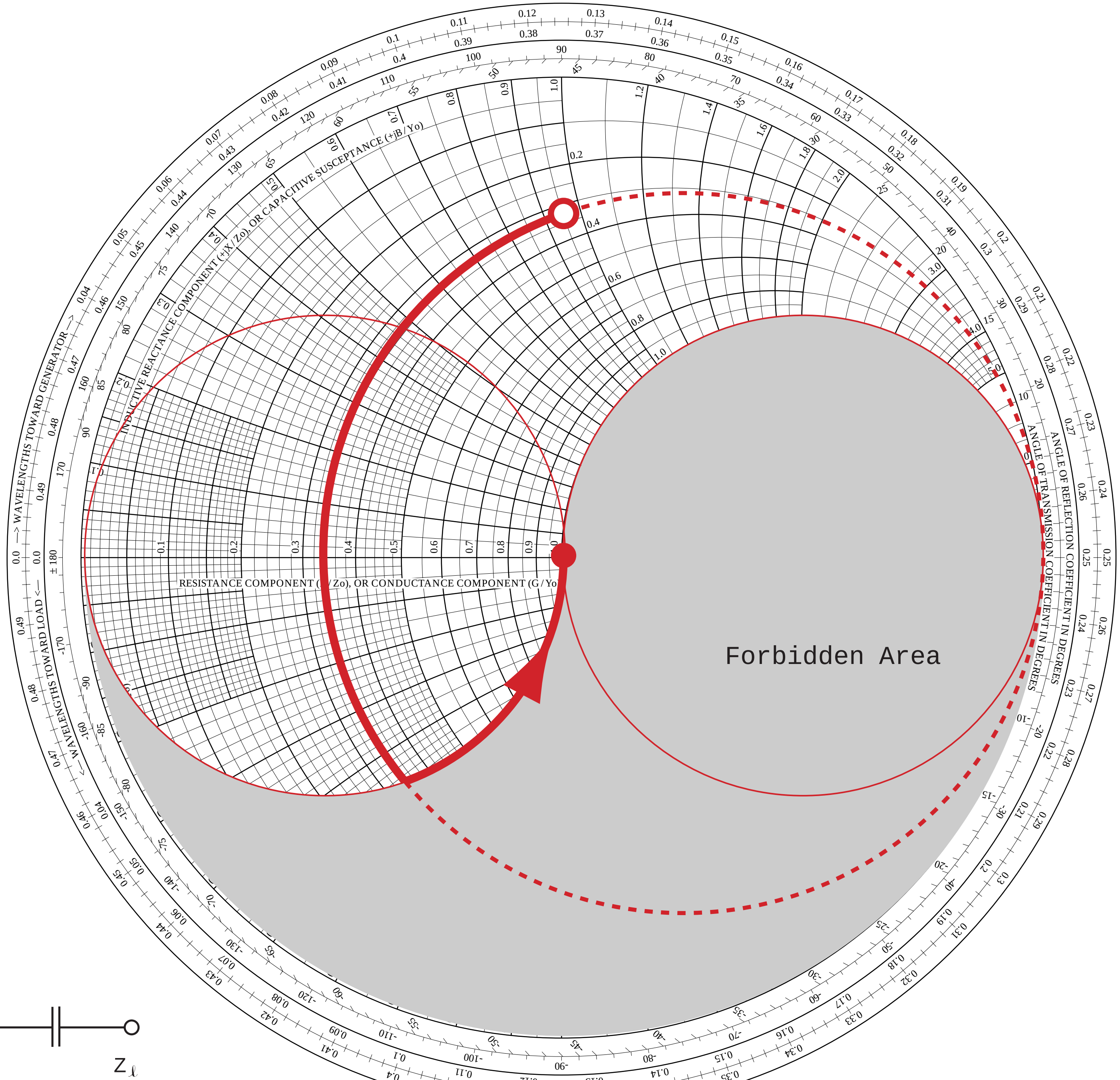


Forbidden Area
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)

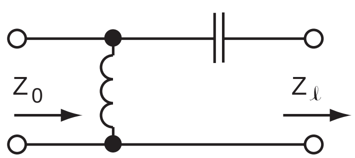


Z_0
Z_ℓ

(c)

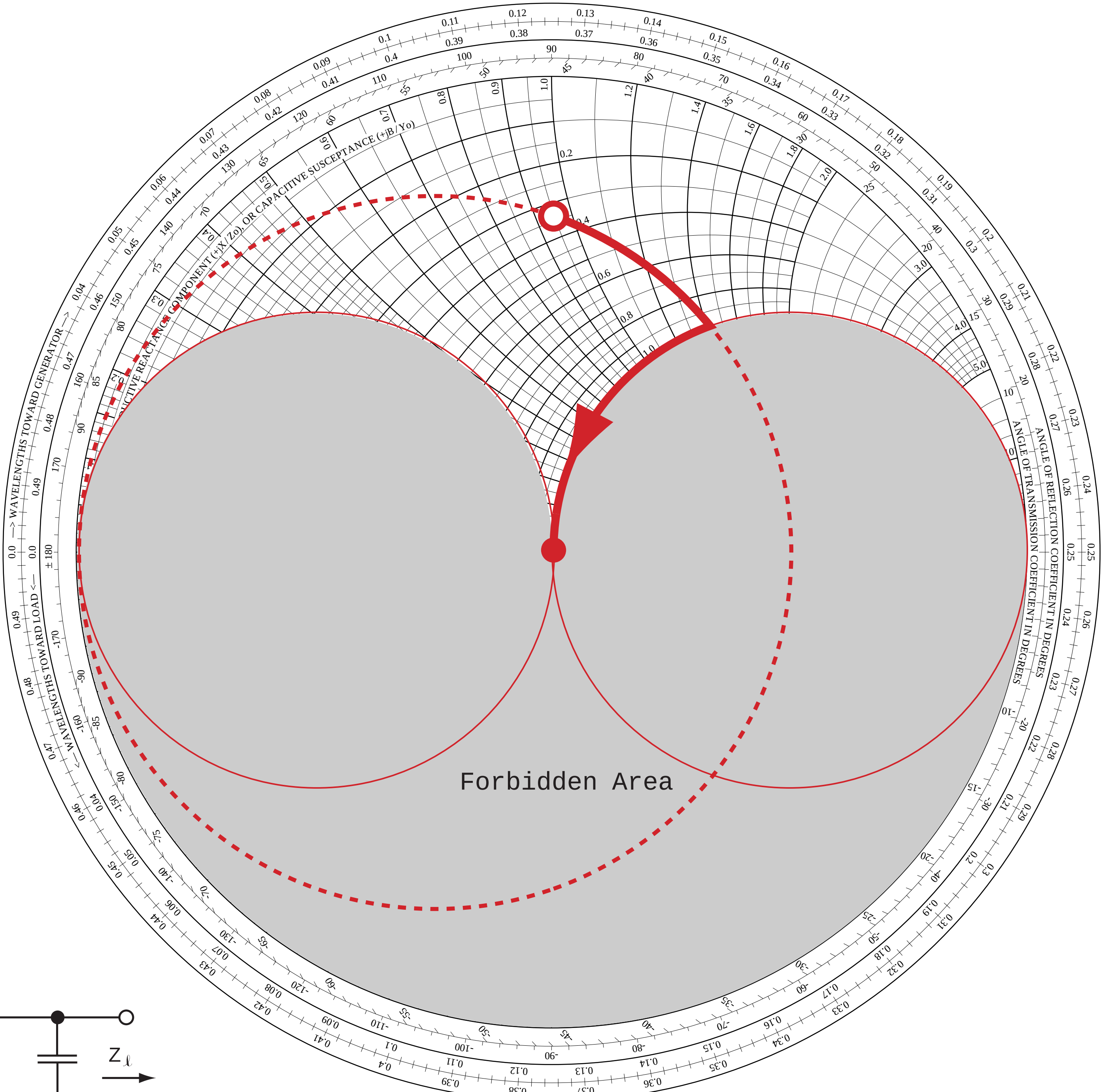


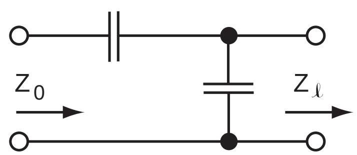
Forbidden Area
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES



Z_0
Z_ℓ

(d)





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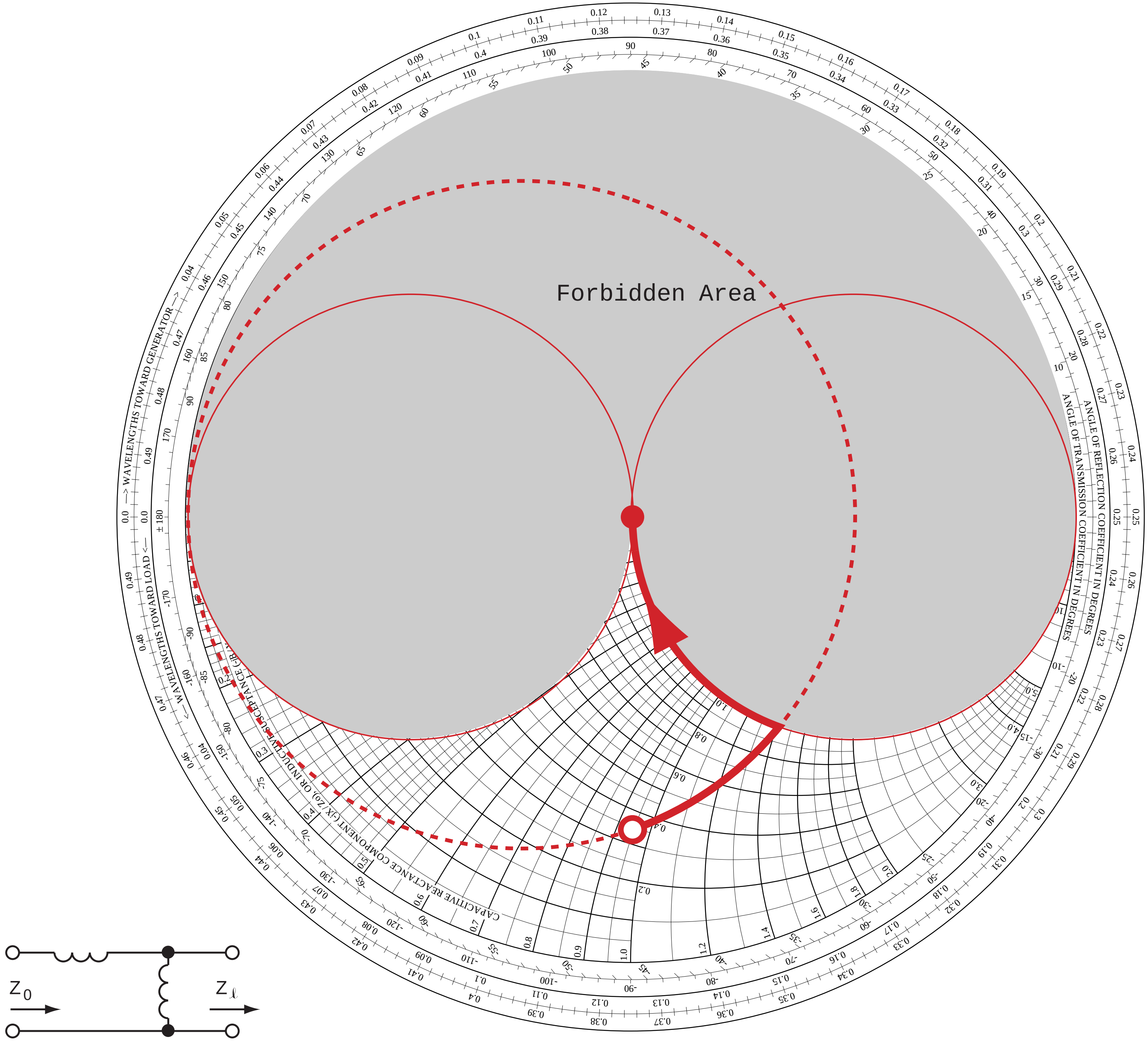

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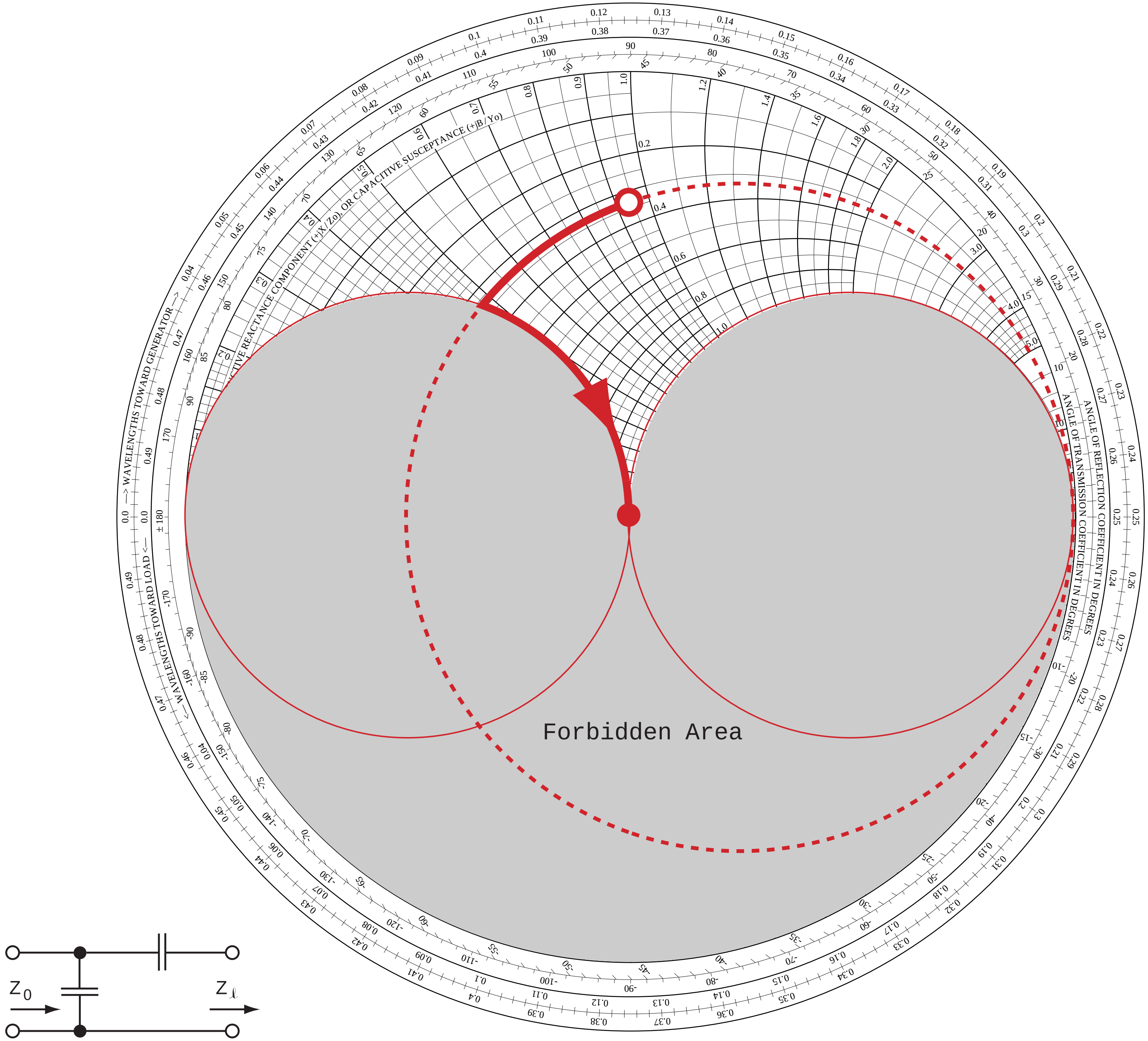

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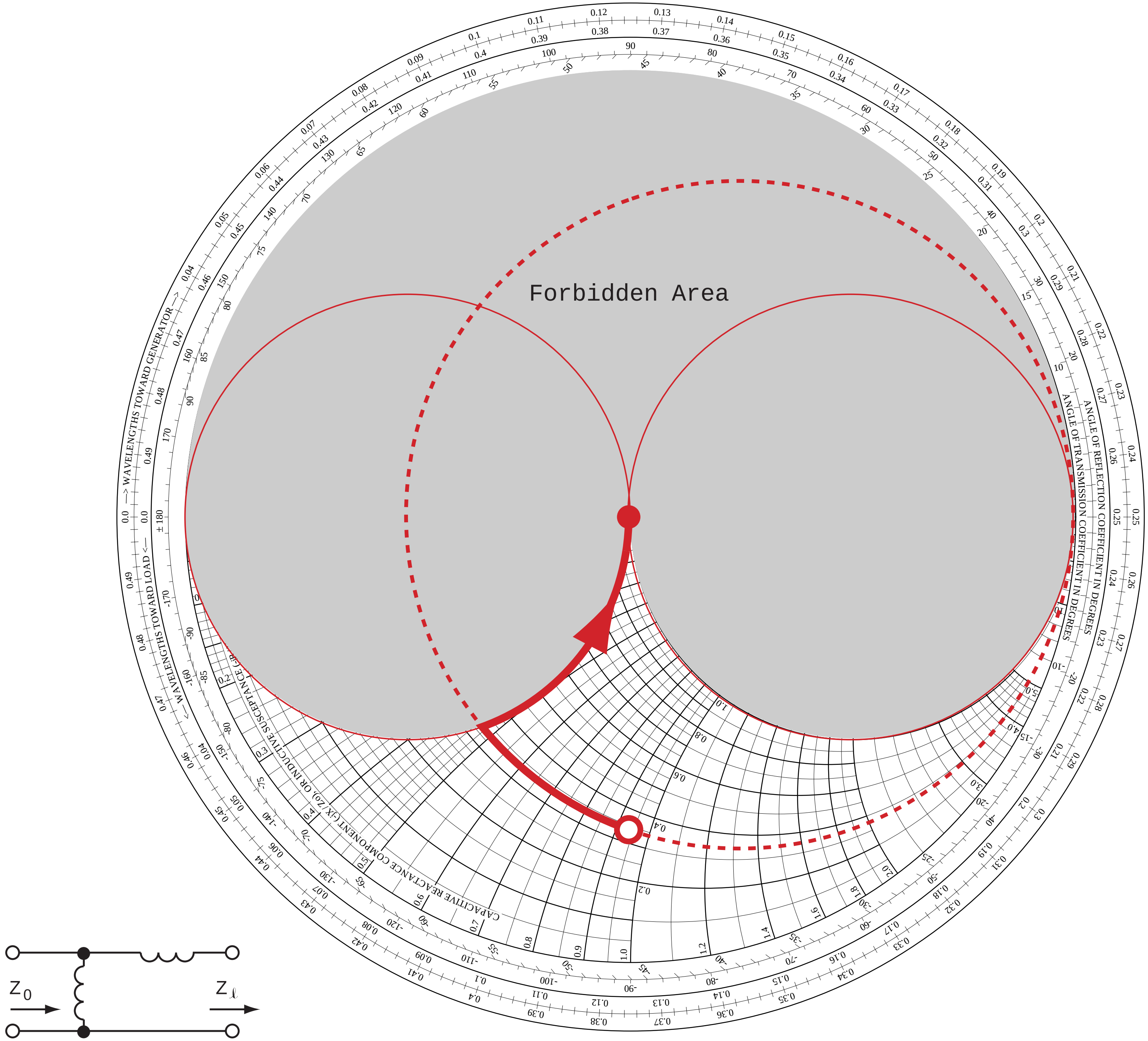
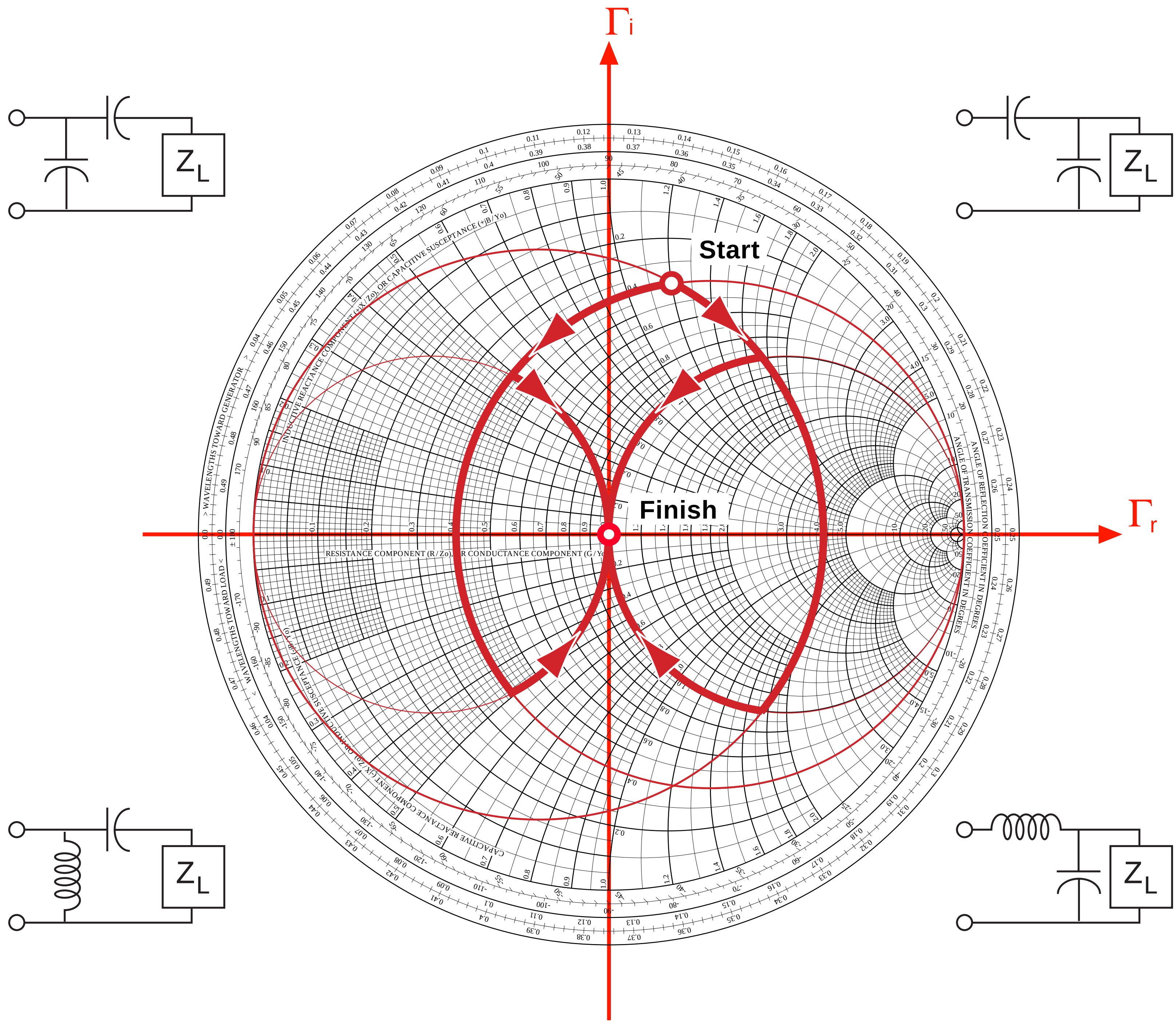
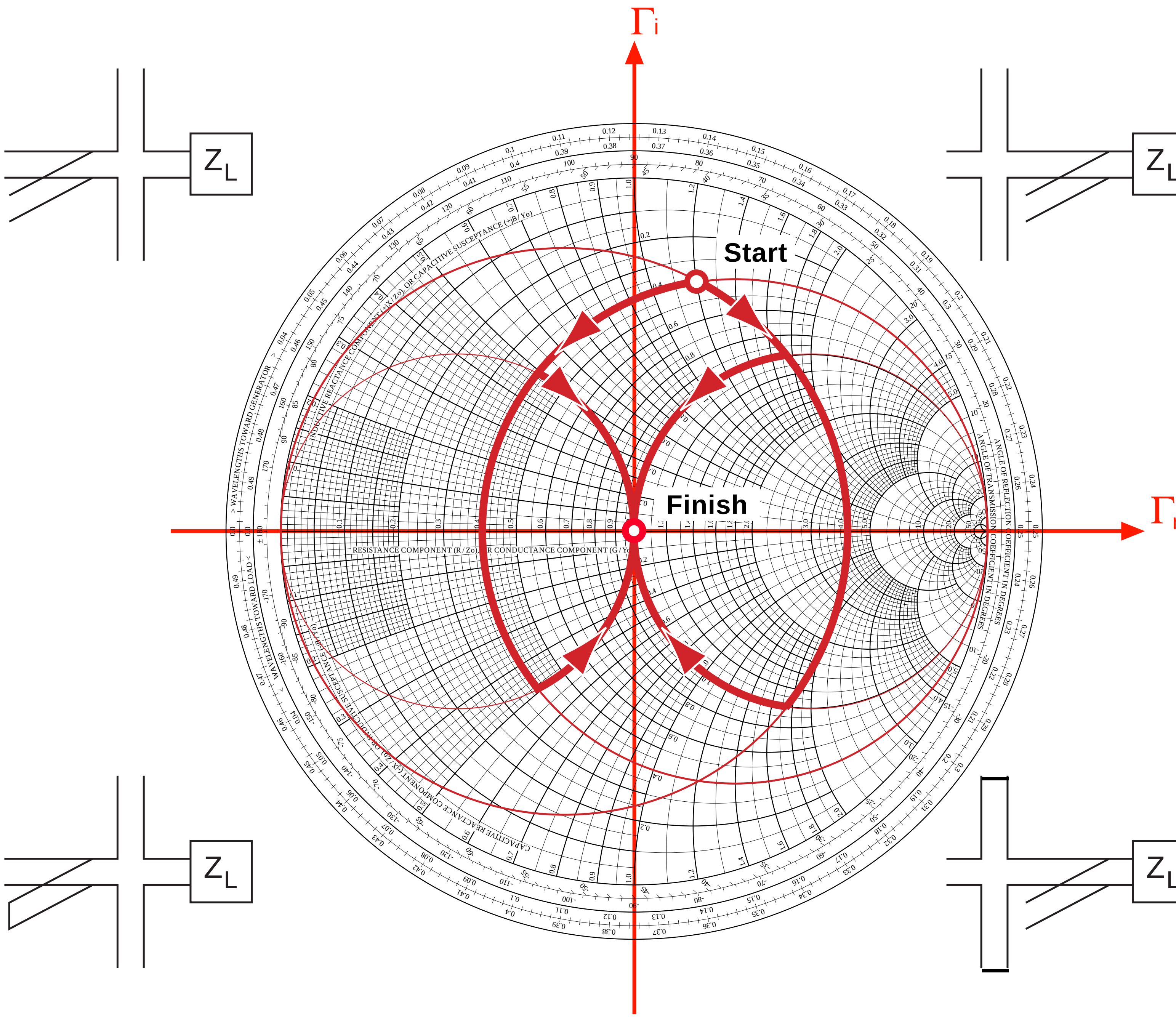


(h)

Matching: Four L-Networks Using Lumped Elements



Matching: Four L-Networks Using Stubs



Reactance & Susceptance of Lumped Elements

Inductors

$$X_L = 2\pi f L \qquad B_L = \frac{-1}{2\pi f L}$$

Capacitors

$$X_C = \frac{-1}{2\pi f C} \qquad B_C = 2\pi f C$$

Reactance & Susceptance of Stubs

- ***Assuming Zo is real, then***
- ***Shorted Stubs***

$$X_{shorted} = Z_o \tan\left(\frac{2\pi lf}{v_f c}\right) \qquad B_{shorted} = \frac{-1}{Z_o \tan\left(\frac{2\pi lf}{v_f c}\right)}$$

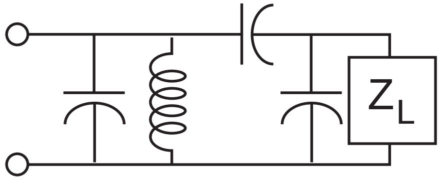
- ***Open Stubs***

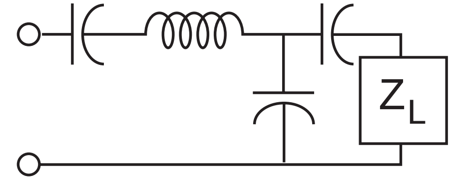
$$X_{open} = \frac{-Z_o}{\tan\left(\frac{2\pi lf}{v_f c}\right)} \qquad B_{open} = \frac{1}{Z_o} \tan\left(\frac{2\pi lf}{v_f c}\right)$$

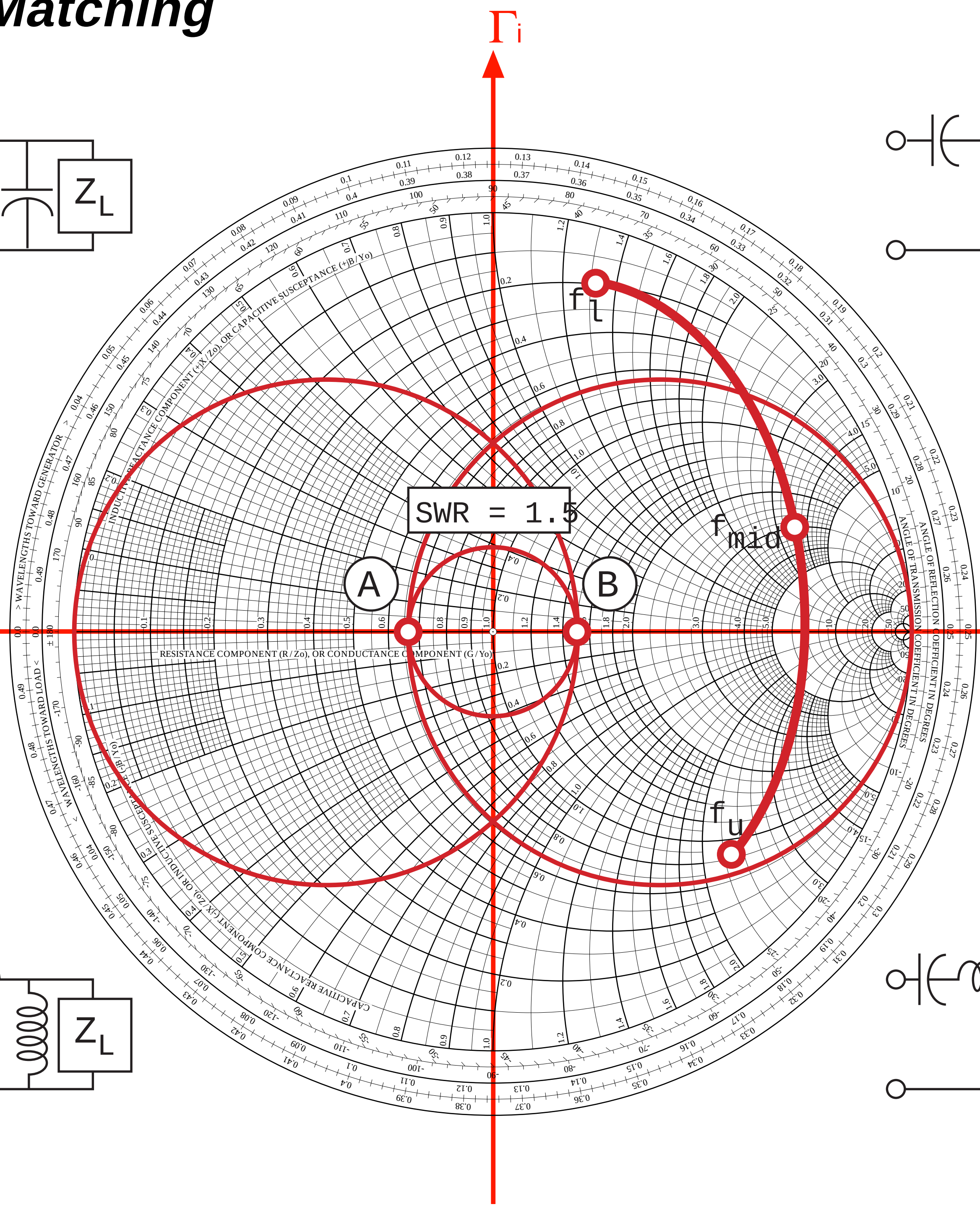
Broadband Matching Network Design Recipe
Using 4-Element/π-Resonant and T-Resonant Networks

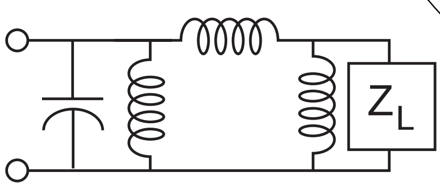
- ***Putting network insertion point close to load (antenna) gives greatest SWR bandwidth***
- ***Step 1: Using an L-network, move the midband impedance point to prime point A or B. Bandwidth will be maximized if the minimum reactance or susceptance L-network is chosen in this step.***
- ***Step 2: Wrap the impedance locus into the SWR circle by adding a series or parallel resonant circuit as required to complete the π-resonant or T-resonant network***

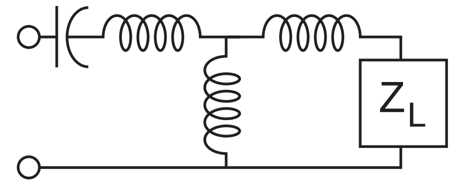
Π - Resonant and T-Resonant Moderate and Broadband Matching



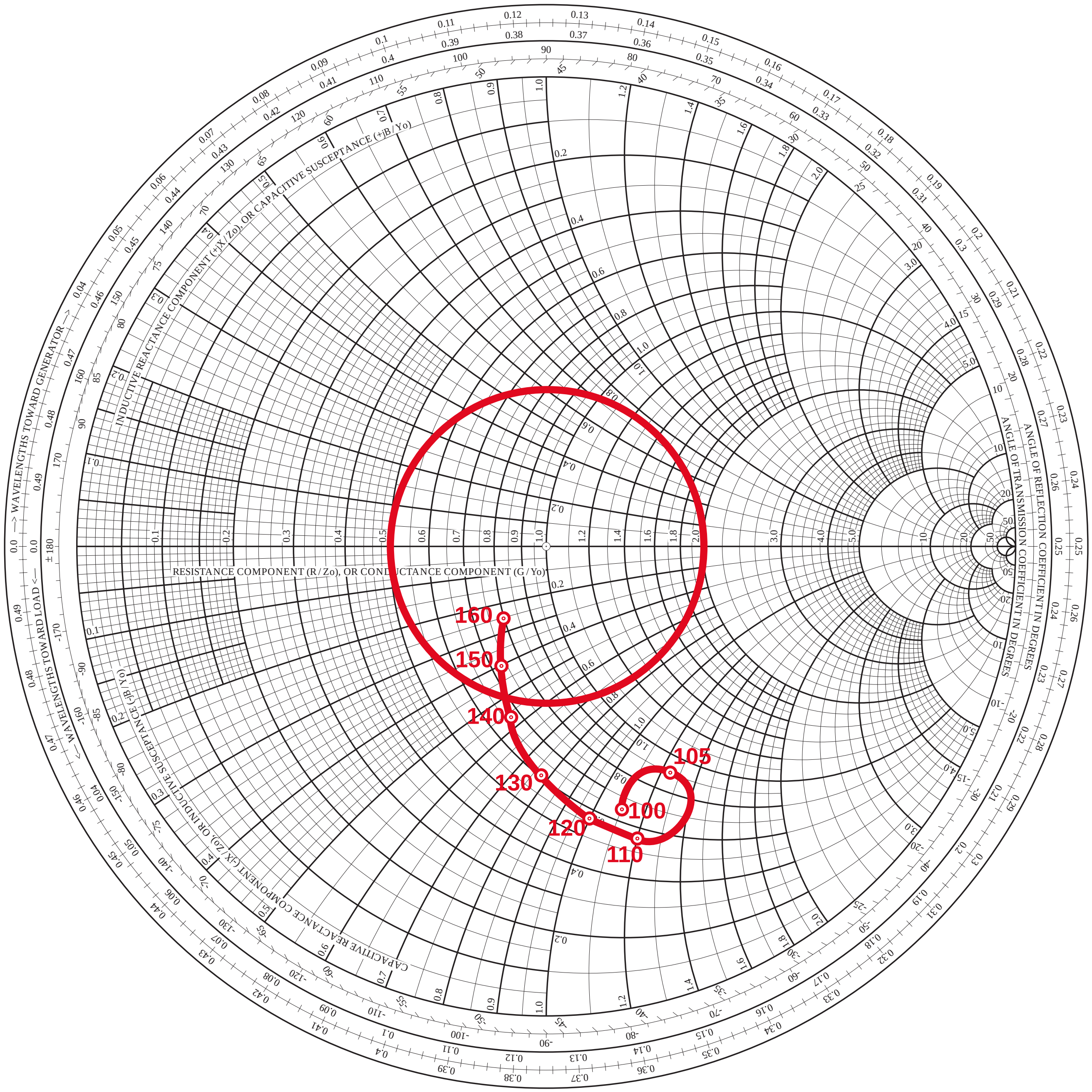




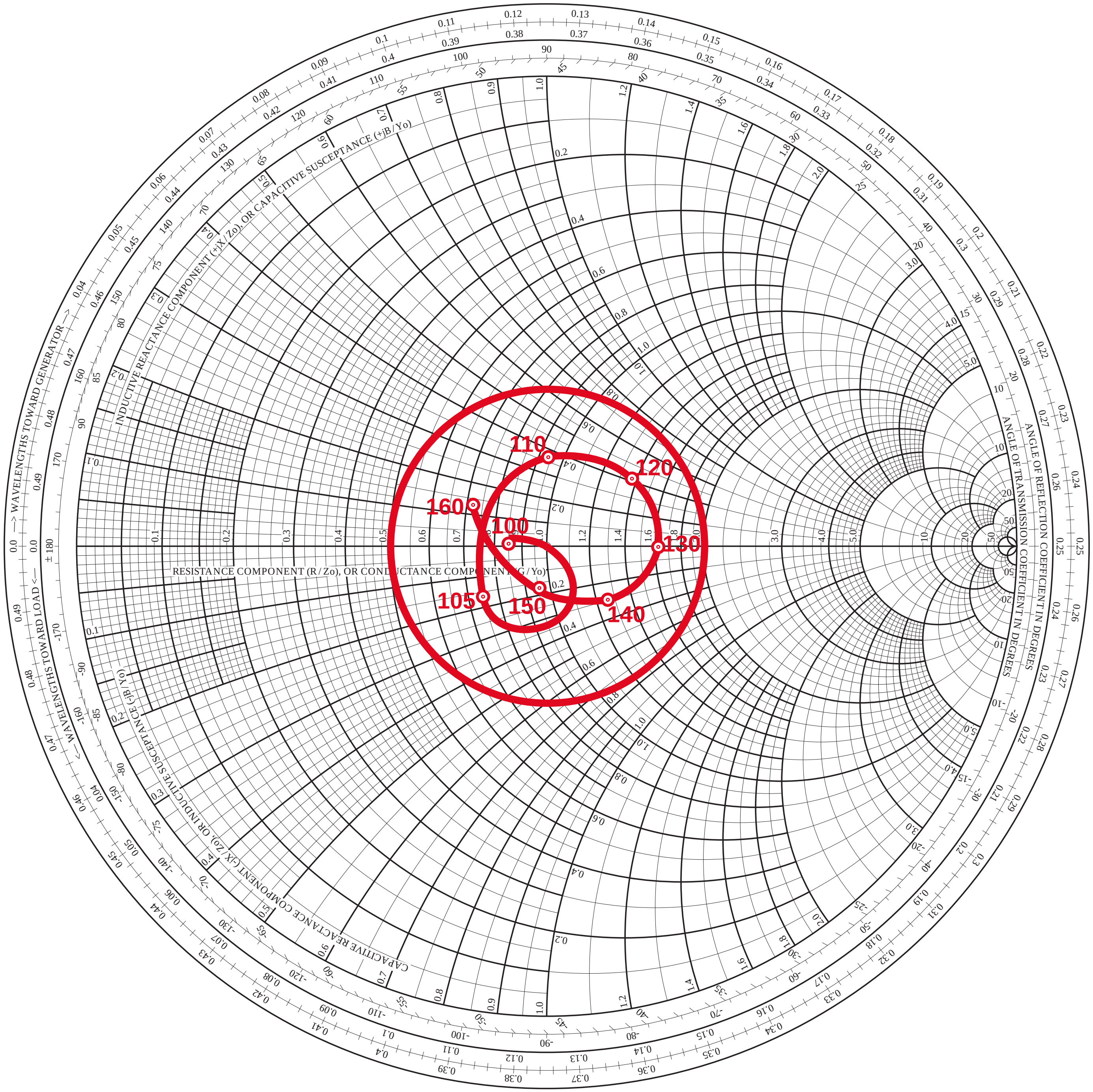




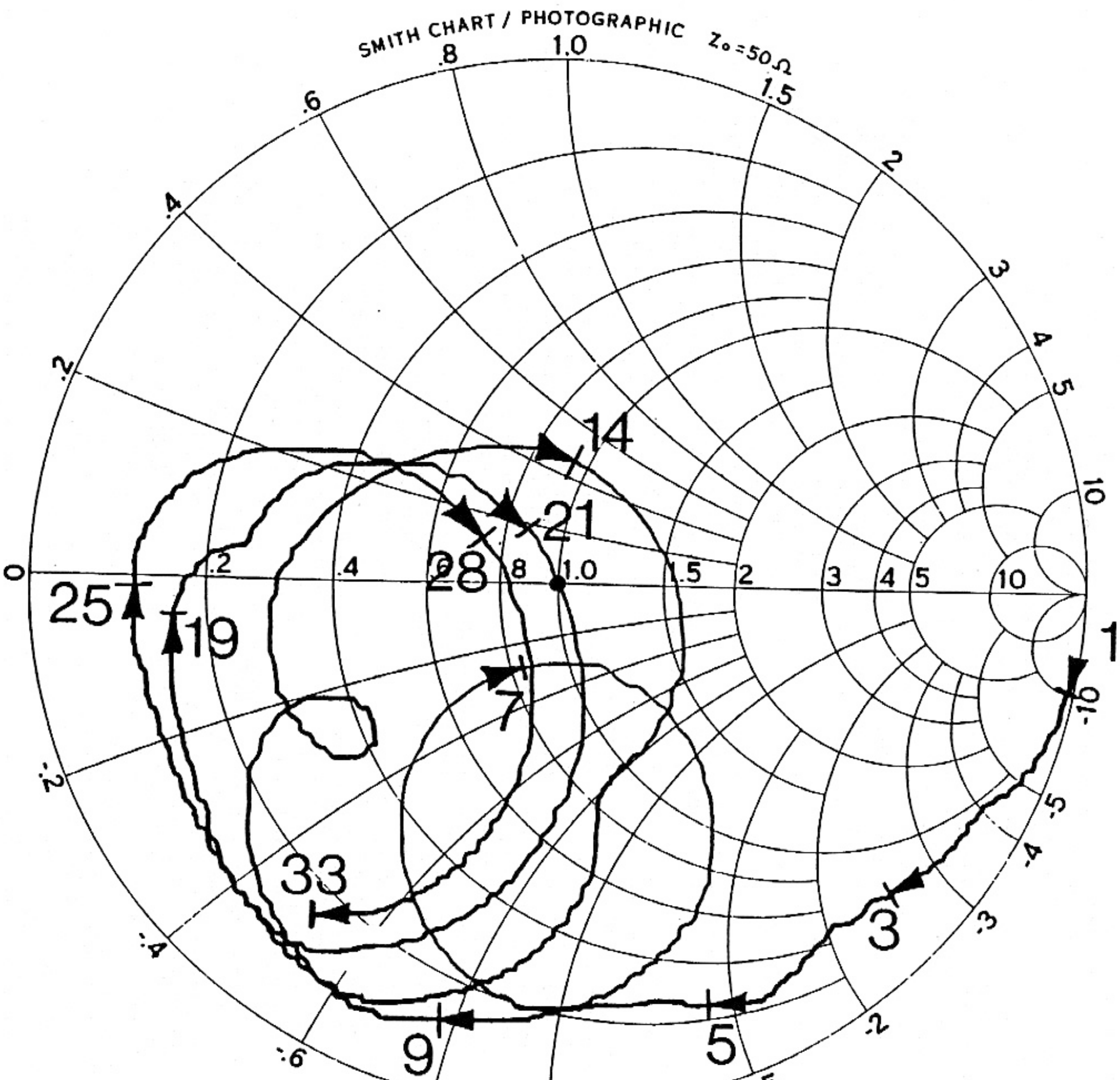
Example A: Moderate Band Matching Problem



Example A: Results of Narrowband Matching



Example B: Multiple Frequency Antenna



Smith Chart Software

Smith Chart Analysis & Display

***MicroSmith* 2.3, ARRL, 1992, $39**

***winSMITH* 2.0, Noble Publishing, 1995, $79**

Matching Network Optimization & Synthesis

***Radio Designer* 1.5, ARRL, 1995, $150**

***Advanced Automated Smith Chart* 3.0, Artech House, 1998, $395**

***=MATCH=,* Eagleware, $699 (requires *GENESYS Basic*, $1,997)**

References: Articles

Neal C. Silence, "The Smith Chart and its Usage in RF Design," *RF Design*, pp. 85-88, April, 1992.

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Steve Sparks, "A Practical Amateur Application of the Smith Chart," *Communications Quarterly*, pp. 102-106, Summer 1999

References: Books

Robert A. Chipman, *Transmission Lines*, Schaum Outline Series, McGraw-Hill, 1968. Basic, mathematical.

Robert L. Thomas, *A Practical Introduction to Impedance Matching,* Artech House, 1976, ISBN 0-89006-050-9. Intermediate, mathematical.

Pieter L. D. Abrie, *The Design of Impedance-Matching Networks for Radio-Frequency and Microwave Amplifiers,* Artech House, 1985, ISBN 0-89006-172-6. Advanced, mathematical.

Wilfred Caron, *Antenna Impedance Matching*, ARRL, 1989, ISBN 0-87259-220-0. Intermediate, non-mathematical.

Phillip H. Smith, *Electronic Applications of the Smith Chart in Waveguide Circuit, and Component Analysis*, 2nd edition, Noble Publishing, 1995, ISBN 1-884932-39-8. Intermediate, mathematical

***ARRL Antenna Book*, 18th edition, chapter 28, ARRL, 1997, ISBN 0-87259-613-3. Elementary, non-mathematical.**

References: Videos and Web Sites

Videos

Glenn Parker, *Introduction to the Smith Chart*, Noble Publishing, 1996. 50 minutes, $99.

Web Sites

http://www.sss-mag.com/smith.html

http://www.ee.surrey.ac.uk/Personal/D.Jefferies/smith.html

http://www.scott-inc.com/html/smith.htm

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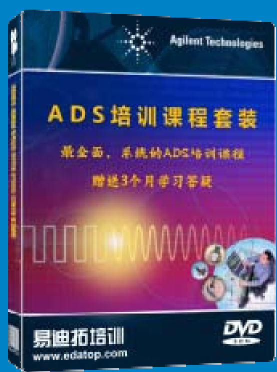

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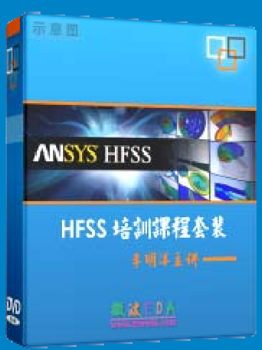

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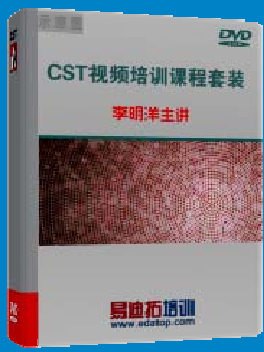

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