Lecture #10 : Basic RF parameters and equations

1. NF
   - Noise source
   - Definition of noise figure
   - Noise figure in a noisy two-port block
   - Minimum of NF and equivalent R_n

2. Gain
   - Definition of power gains
   - Power gain and voltage gain

3. Sensitivity of a receiver
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   - Equivalent input noise
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4. Non-linearity and spurious products
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5. Distortion

6. Cascaded equations
   - Cascaded equation for power gain
   - Cascaded equation for noise figure
   - Cascaded equation for intercept point
   - Application of cascaded equation system analysis

7. From analog to digital communication system
   - Modulation in an analog communication system
   - Encoding in a digital communication system
   - Decoding and bit-error probabilit
   - Error correction schemes

1. Noise Figure

o Noise sources

* Shot noise

- Shot noise is associated withthe current itself.

- A current appears to be stable in macrocosm whereas it appears to be fluctuated in microcosm.

- A current, in fact, is composed of a large number of moving carriers, either positive charger or electron with random velocity. The product of electric charge and velocity of a charger or a electron forms a current element.

- Owing to the different velocity these current elements are different from each other. The sum of all the current elements at a junction of semiconductor or a cross-section of a part looks like a large number of current pulses. It is slightlydifferent from time to time due to different number of current pulses and different velocities between them. The average value of these random current pulsesis called current and its fluctuation around the average value is called shot noise.

$$\overline{i_n^{\ 2}} = 2qI\Delta f$$

where i_n = Current fluctuation,
I = Average value of current,
q = Charge of an electron $=1.6 \times 10^{-19}$ Columb,
Δf = Bandwidth in which noise source is acting.

* Thermal noise

- **Thermal noise is essentially the fluctuation of the resistance in the parts or devices.**
- **The electron plays the main role in the formation of resistance because the electron has much higher mobility than other charge carriers. The collision between electrons and between electron and other charge carriers or particles forms the resistance. The random thermal motion and collision of the electron produce the thermal noise while the random drift velocity of the moving charge-carriers or the current fluctuation produces the shot noise. The velocity of the electron thermal motion is much higher than the drift velocity of the electron and other charge-carriers.**
- **Therefore, the thermal noise is a completely different mechanismfrom shot noise. The thermal noise is still existed even when the shot noise is not existed due to zero current.**

The thermal noise of a resistor, R, can be represented by eithervoltage or current noise source, that is,

$$\overline{e_n^{\,2}} = 4kTR\ \Delta f$$

or,

$$\overline{i_n^{\,2}} = 4kT\frac{1}{R}\Delta f$$

where e_n **= Voltage fluctuation,**
i_n **= Current fluctuation,**
k **= Boltzmannconstant,**
T **= Room temperature,**
R **= Resistance of resistor,**
Δf **= Bandwidth in which noise source is acting.**

* Flicker noise

- **At the early stage, flicker noise is found in the emitter-base depletion layer due to the contamination and crystal defects. The traps capture and releasecarriers in a random fashion associated a noise with energy concentrated at low frequency.**

- **Later on, it is found in all the active devices as well as in some passive parts. It always only exists in associated with a direct current. Therefore, a resistor carrying not direct current does not have flicker noise. A metal type of external resistor has not flickernoise, because the traps associated with contamination is not existed. It is therefore recommended if the direct current is to be carried.**

Flicker noise can be represented by a noise current source as follows:

$$\overline{i_n^2} = k_f \frac{I^a}{f^b} \Delta f$$

where i_n **= Current fluctuation,**
k_f **= A constant for a particular devise,**
I = Direct current,
a = 0.5 to 2,
b $\approx$ 1

Flicker noise is significant in the low frequencies. Therefore in PLL and oscillator design, the Flicker noise is one of the main sources of the phase noise.

o Concept of noise figure

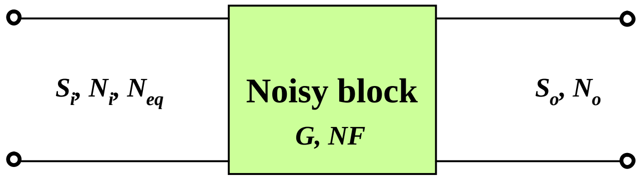


Figure 10.1 The various parameters related to the definition ofnoise figure.

$$\frac{S_o}{N_o} < \frac{S_i}{N_i}$$

$$S_o = GS_i$$

$$N_o = GN_i + \Delta N$$

$$\Delta N > 0$$

where Δ N = additional noise power produced by the circuit block,
S_i, S_o = Signal power at input and output respectively,
N_i,N_o = Noise power at input and output respectively,
G = power gain of the circuit block.

o Definition of noise figure

$$NF = \frac{\dfrac{S_i}{N_i}}{\dfrac{S_o}{N_o}} = \frac{N_o}{GN_i}$$

$$NF = \frac{N_o}{GkT\Delta f} = \frac{N_{eq}}{kT\Delta f}$$

$$\boxed{NF = \frac{\text{Total output noise power}}{\text{Output noise power due to the input noise}}}$$

where

S_i, S_o = Signal power at input and output respectively;
N_i,N_o = Noise power at input and output respectively;
N_{eq} = Equivalent input noise of output noise.

$$G = \frac{S_o}{S_i}$$

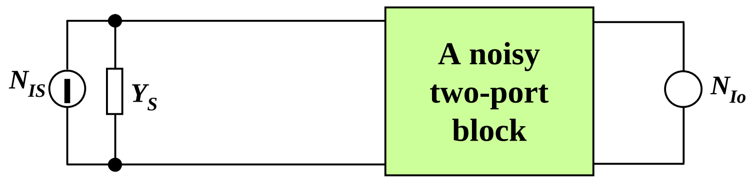
$$N_i = kT\Delta f$$

$$N_{eq} = \frac{N_o}{G}$$

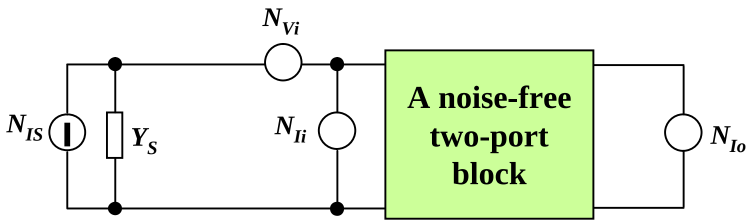
o Noise figure in a noisy two-port block

H.A.Haus(Chairman of IRE sub-committee 7.9 on Noise) et.al., "Representation of noise in Linear Twoports", Proceedings of the IRE, p.69, January 1960.

* Input noise generators



(a) Characterization of a noisy two port block by source noise current, output noise current, and a noisy two-port block.



(b) Characterization of a noisy two port block by source noisecurrent, input noise voltage and noise current, output noise current, and a noise-free two-port block.

Figure 10.2 Noise characterization of a noisy two-port block.

* Expression of noise figure

The output noise now contributed by three noise generators, thatis

$$\overline{i_{no}^{\ 2}} = G\left(\overline{i_{ns}^{\ 2}} + \overline{|\, i_{ni} + Y_s e_{ni}\,|^2}\right)$$

where G = power gain of the noisy two-port block.
The output noise due to the source noise only is

$$\left.\overline{i_{no}^{\ 2}}\right|_{n.s.only} = G\left(\overline{i_{ns}^{\ 2}}\right)$$

Consequently, from the two expressions above, the noise figure of the noisy two-port block is

$$NF = \frac{\overline{i_{no}^{\ 2}}}{\overline{i_{n.s.only}^{\ 2}}} = \frac{\overline{i_{ns}^{\ 2}} + \overline{|\, i_{ni} + Y_s e_{ni}\,|^2}}{\overline{i_{ns}^{\ 2}}} = 1 + \frac{\overline{|\, i_{ni} + Y_s e_{ni}\,|^2}}{\overline{i_{ns}^{\ 2}}}$$

In the denominator, the mean square of source noise current fluctuation is related to the source conductance, G_S, by the Nyquistformula:

$$\overline{i_{ns}^{\ 2}} = 4\,kTG_S\,\Delta f$$

In the numerator, i_{ni} must be splittedinto two components:
One is perfectly correlated and another one is uncorrelated withthe input noise voltage fluctuation, e_{ni}, that is,

$$i_{ni} = \left(i_{ni} - i_{ni,u}\right) + i_{niu}$$

where i_{niu} = Uncorrelated component of i_{ni} with e_{ni}, the remained component, (i_{ni}-i_{niu}), of course, is correlated with e_{ni}.

* Correlated and un-correlated

The correlated component, (i_{ni}-i_{niu}), is correlated to e_{ni} and can be expressed by a proportional constant, Y_c.

$$i_{ni} - i_{niu} = Y_c e_{ni}$$

$$\overline{\left| i_{ni} - i_{niu} \right|^2} = Y_c^{\ 2}\,\overline{e_{ni}^{\ 2}}$$

where Y_c = correlated admittance of Y_s with e_{ni}, and is

$$Y_c = G_c + jB_c$$

where G_c = Correlated conductance of the source admittance,
B_c = Correlated susceptanceof the source admittance.
Similarily, the mean square of uncorrelated input noise current can be expressed as

$$\overline{i_{ni,u}^{\ 2}} = 4kTG_u \Delta f$$

$$\overline{e_{ni}^{\ 2}} = 4kTR_n \Delta f$$

where G_u = Uncorrelated conductance of the source admittance.
The input noise voltage can be expressed in terms of an equivalent noise resistance, R_n,

* **Mathematic operation**

$$\overline{|i_{ni}+Y_s e_{ni}|^2} = \overline{(i_{ni}+Y_s e_{ni})(i_{ni}^*+Y_s^* e_{ni}^*)} = \overline{i_{ni}^2 + Y_s^2 e_{ni}^2 + e_{ni} i_{ni}^* Y_s + e_{ni}^* i_{ni} Y_s^*}$$

$$\overline{|i_{ni}+Y_s e_{ni}|^2} = \overline{(i_{ni}-i_{niu})^2} + \overline{i_{niu}^2} + Y_s^2\overline{e_{ni}^2} + Y_c^* Y_s \overline{e_{ni}^2} + Y_c Y_s^* \overline{e_{ni}^2}$$

$$\overline{|i_{ni}+Y_s e_{ni}|^2} = Y_c^2\overline{e_{ni}^2} + \overline{i_{niu}^2} + Y_s^2\overline{e_{ni}^2} + Y_c^* Y_s \overline{e_{ni}^2} + Y_c Y_s^* \overline{e_{ni}^2}$$

$$\overline{|i_{ni}+Y_s e_{ni}|^2} = \overline{i_{niu}^2} + |Y_s + Y_c|^2\,\overline{e_{ni}^2}$$

In the process of the above derivation, the following relationshave been applied:

$$\overline{e_{ni}^* i_{niu}} = \overline{e_{ni} i_{niu}^*} = 0$$

$$\overline{e_{ni}^* i_{ni}} = \overline{e_{ni}^*(i_{ni}-i_{niu})} = Y_c\overline{e_{ni}^2}$$

$$\overline{e_{ni} i_{ni}^*} = \overline{e_{ni}(i_{ni}-i_{niu})^*} = Y_c^*\overline{e_{ni}^2}$$

$$(A+B)^* = A^* + B^*$$

$$\overline{A+B} = \overline{A} + \overline{B}$$

$$AA^* = |A|^2$$

$$(AB)^* = A^* B^*$$

Finally the noise figure, NF, becomes

$$NF = 1 + \frac{G_u}{G_s} + \frac{R_n}{G_s}\left[(G_s + G_c)^2 + (B_s + B_c)^2\right]$$

* Minimization

From the minimized condition, by differentiating of equation inrespect to B_s,

$$\frac{\partial(NF)}{\partial B_s} = 0 \qquad B_o = B_s = -B_c$$

where B_o = Optimum value of source susceptance, B_s, which enables NF to reach a minimum.
The equation becomes

$$NF = 1 + \frac{G_u}{G_s} + \frac{R_n}{G_s}\left(G_s + G_r\right)^2$$

Now, from another minimized condition, by differentiating of equation in respect to G_s,

$$\frac{\partial(NF)}{\partial G_s} = 0 \qquad \frac{\partial(NF)}{\partial G_s} = \frac{G_s 2R_n\left(G_s + G_c\right) - G_u - R_n\left(G_s + G_c\right)^2}{G_s^{\ 2}} = 0$$

$$G_o = G_s = \sqrt{\frac{G_u + R_n G_c^{\ 2}}{R_n}}$$

$$NF_{\min} = 1 + 2R_n\left(G_c + G_o\right)$$

Finally,

$$NF = NF_{\min} + \frac{R_n}{G_s}\left[\left(G_s - G_o\right)^2 + \left(B_s - B_o\right)^2\right]$$

* A remarkable significance.

A minimum of noise figure is existed as long as the source admittance is adjusted to its optimum values,

$$Y_s = Y_o$$

Where

$$Y_s = G_s + jB_s,$$

$$Y_o = G_o + jB_o.$$

* 4 points method of noise figure testing.

- **Noise figure is a function of another 4 parameters, NF_{min}, R_n, G_o, and B_o.**
- **By means of 4 times testing of noise figure with 4 different source admittances, 4 equations can be established.**
- **Consequently the 4 unknown parameters, NF_{min}, R_n, G_o, and B_o can be solved.**

o Minimum of *NF* and equivalent R_n

* MOSFET device

The drain noise current and gate noise current. Their mean-square can be expressed by

$$\overline{i_{nd}^{\,2}} = 4kT\gamma g_{do}\Delta f$$
$$\overline{i_{ng}^{\,2}} = 4kT\delta g_{g}\Delta f$$

respectively,

where i_{nd} = Drain current noise fluctuation,

i_{ng} = Gate current noise fluctuation,

γ = 1 for short channel device at V_{DS}=0 and decreases toward 2/3 for long channel device in saturation

δ = Coefficient to be about twice of γ.

g_{do} = Drain-source conductance at zero V_{DS},

g_g = Gate-source conductance and is

$$g_g = \frac{w^2 C_{gs}^{\,2}}{5 g_{do}}$$

The gate noise is correlated with the drain noise with a correlation coefficient, c

$$c = \frac{\overline{i_{ng} \cdot i_{nd}^{*}}}{\sqrt{\overline{i_{ng}^{\,2}} \cdot \overline{i_{nd}^{\,2}}}}$$

***c* is about j0.395 for a long channel device.**

The first equivalent noise voltage generator, e_{ni}, accounts for the equivalent input noise of output noise when the input is short-circuited. It is an equivalent noise voltage reflected from the drain noise current via g_m, that is,

$$e_{ni} = \frac{i_{nd}}{g_m}$$

$$\overline{e_{ni}^{2}} = \frac{\overline{i_{nd}^{2}}}{g_m^{2}} = \frac{4kT\gamma g_{do}\Delta f}{g_m^{2}}$$

Obviously, i_{nd} is correlated with e_{ni} by the correlation coefficient g_m. It corresponds to an input noise resistor, R_n,

$$R_n = \frac{\overline{e_{ni}^{2}}}{4kT\Delta f} = \frac{\gamma g_{do}}{g_m^{2}}$$

The second noise generator, the input noise current fluctuation, i_{ni}, accounts for the equivalent input noise of output noise when the input is open-circuited. It consists of two parts:

The first part is the equivalent noise voltage reflected from the drain noise current via g_m, which is correlated with e_{ni} by the correlation coefficient g_m and forms the input noise current to the input correlated admittance Y_c. It is a good approximation to assume the input admittance of aMOSFET is purely capacitive. Then,

$$i_{ni,1} = \frac{i_{nd}\, jwC_{gs}}{g_m} = e_{ni}\, jwC_{gs}$$

The second part is the input noise current resulted from the induced gate noise current fluctuation. However, only those portions, which is correlated with the drainnoise current fluctuation, i_{ngc}, contributes the noise to the input noise current generator, i_{ni}. Consequently,

$$i_{ni,2} = i_{ngc}$$

The correlated input admittance is

$$Y_c = \frac{i_{ni,1} + i_{ni,2}}{e_{ni}} = jwC_{gs} + \frac{i_{ngc}}{e_{ni}} = jwC_{gs} + g_m \frac{i_{ngc}}{i_{nd}}$$

$$Y_c = jwC_{gs} + g_m c\sqrt{\frac{\overline{i_{ng}^{\,2}}}{\overline{i_{nd}^{\,2}}}}$$

if c is a pure imaginary. Then, it results that

$$Y_c = jwC_{gs} + \frac{g_m}{g_{do}} c\sqrt{\frac{d}{5g}} wC_{gs} = jwC_{gs}\left(1 + \frac{g_m}{g_{do}}|c|\sqrt{\frac{d}{5g}}\right)$$

$$G_c = 0$$

$$B_c = wC_{gs}\left(1 + \frac{g_m}{g_{do}}|c|\sqrt{\frac{d}{5g}}\right)$$

The last one of 4 noise figure parameters, G_u, can be derived from the uncorrelated gate noise current

$$G_u = \frac{\overline{i_{ngu}^{\,2}}}{4kT\Delta f} = \frac{4kT\Delta f d g_g \left(1 - |c|\right)^2}{g_m^{\,2}} = \frac{dw^2 C_{gs}^{\,2}\left(1 - |c|^2\right)}{5 g_{do}}$$

The optimum source conductance, G_o, is

$$G_o = \sqrt{\frac{G_u}{R_n} + G_c^{\,2}} = \frac{g_m}{g_{do}} w C_{gs} \sqrt{\frac{d}{5g}\left(1 - |c|^2\right)}$$

The optimum source susceptance, B_o, is

$$B_o = -B_c = -wC_{gs}\left(1 + \frac{g_m}{g_{do}} |c| \sqrt{\frac{d}{5g}}\right)$$

Finally, we have

$$NF_{\min} = 1 + 2R_n\left(G_c + G_o\right) = 1 + \frac{2}{5}\frac{w}{w_T}\sqrt{gd\left(1 - |c|^2\right)}$$

where

$$w_T = \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}}$$

o Minimum of NF and equivalent R_n

* Bipolar device

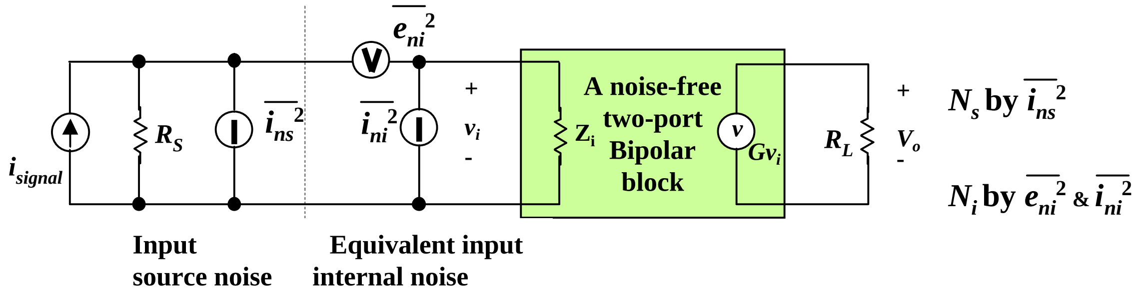


Figure 10.3 Noise figure contributed by source noise and equivalent internal noise

$$G = \frac{V_o}{v_i}$$

$$N_s = \frac{G^2}{R_L} \frac{(R_s Z_i)^2}{(R_s + Z_i)^2} \overline{i_{ns}}^2$$

$$N_i = \frac{G^2}{R_L} \left[\frac{Z_i^2}{(R_s + Z_i)^2} \overline{v_{ni}}^2 + \frac{(R_s Z_i)^2}{(R_s + Z_i)^2} \overline{i_{ni}}^2 \right]$$

$$NF = \frac{N_s + N_i}{N_s} = 1 + \frac{N_i}{N_s} = 1 + \frac{\overline{v_{ni}^2}}{R_s^2 \overline{i_{ns}^2}} + \frac{\overline{i_{ni}^2}}{\overline{i_{ns}^2}}$$

where N_s = Output noise power due to the input source noise current, $\overline{i_{ns}^2}$,
N_i = Output noise power due to the equivalent input internal noisecurrent, $\overline{i_{ns}^2}$,
and the equivalent input internal noise voltage, $\overline{e_{ni}^2}$.

Assuming that

1) **The source impedance is a pure resistor, R_s,**
2) **Only thermal noise and shot noise is considered while the flicknoise and other high frequency effects are neglected,**

then,

$$\overline{i_{ns}^2} = 4kT \frac{1}{R_s} \Delta f$$

$$\overline{v_{ni}^2} = 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f$$

$$\overline{i_{ni}^2} = 2qI_B \Delta f = 2q \frac{I_C}{b_F} \Delta f$$

$$NF = 1 + \frac{\overline{v_{ni}^2}}{4kTR_s \Delta f} + \frac{\overline{i_{ni}^2}}{4kT \frac{1}{R_s} \Delta f}$$

$$NF = 1 + \frac{\overline{v_{ni}^{\;2}}}{4kTR_s\Delta f} + \frac{\overline{i_{ni}^{\;2}}}{4kT\frac{1}{R_s}\Delta f}$$

There is a minimum of NF when

$$R_s = R_{s,opt} = \frac{\overline{v_{ni}^{\;2}}}{\overline{i_{ni}^{\;2}}} = \frac{\sqrt{b_F}}{g_m}\sqrt{1+2g_m r_b}$$

There are 3 parameters, β_F, r_b, and g_m, determine the value of NF_{min}. The higher the β_F, the lower the NF_{min}. The lower r_b, the lower the NF_{min}. The lower g_m, the lower the NF_{min} because g_m is proportional to I_c which bring about the shot noise.

$$NF_{\min} \approx 1 + \frac{1}{\sqrt{b_F}}\sqrt{1 + 2g_m r_b}$$

2. Gain

o Definition of power gains

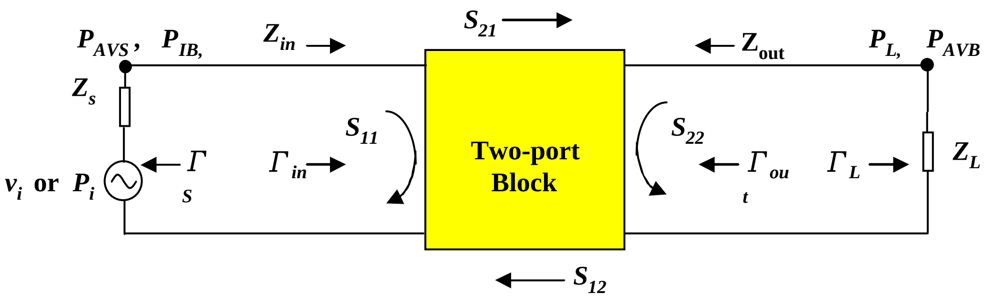


Figure 10.4 Relationship between power gain and related parameters

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{in} - 1}{Z_{in} + 1}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = \frac{Z_{out} - 1}{Z_{out} + 1}$$

To be familiar with signal flow graph!

(Guillermo Gonzalez, Microwave transistor Amplifiers, Analysis and Design, p.80-87, Prentice –Hall, Inc., 1984.)

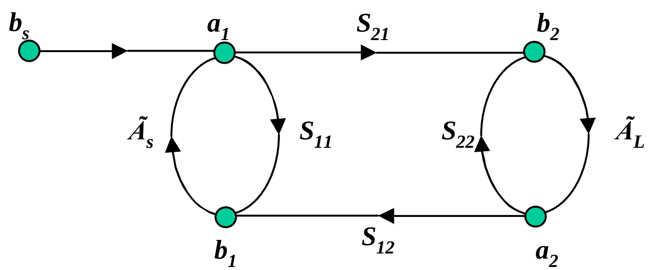


Figure 10.5 Signal flow graph of a two-port block

o Transducer power gain

A transducer power gain is defined as
a ratio of the power delivered to the load, P_L, to the power available from the source, P_{AVS} , that is,

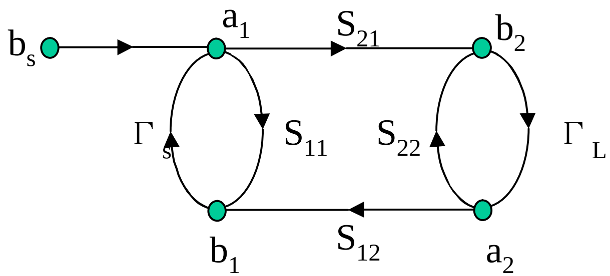
$$G_T = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_s|^2}{|1-\Gamma_{in}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_T = \frac{P_L}{P_{AVS}} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

When $S_{12} = 0$, the block becomes unilateral, that is,

$$\Gamma_{in} = S_{11}$$

$$\Gamma_{out} = S_{22}$$

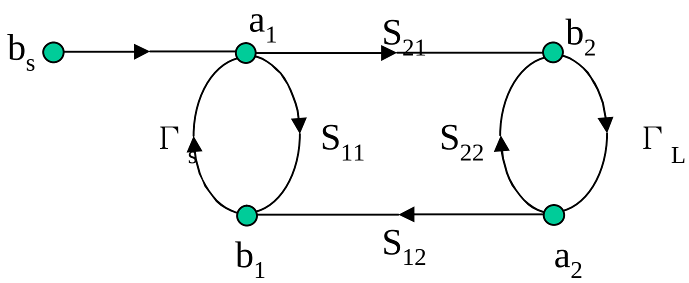


$$G_T = G_s G_o G_L = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}$$

$$G_o = |S_{21}|^2$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$



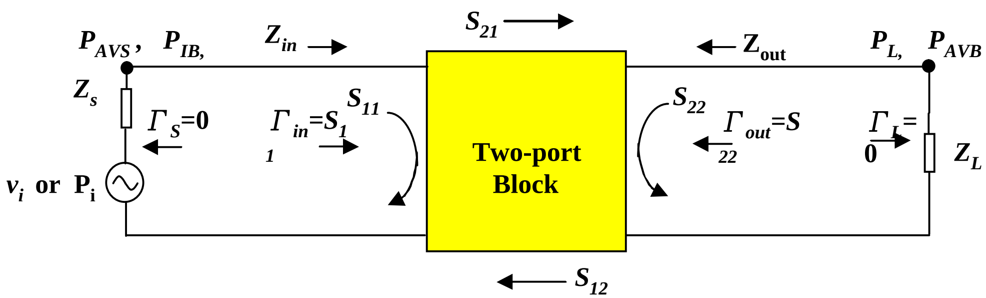
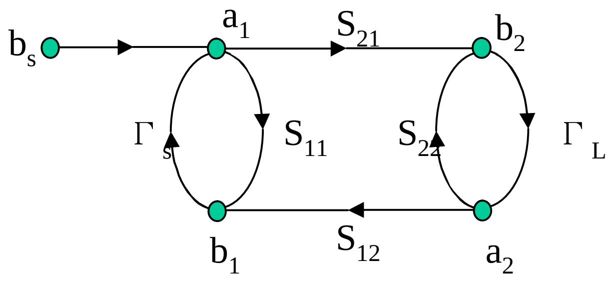


Figure 10.6 Relationship between power gain and related parameters

o Operating power gain (or, Power gain)

An operating power gain (or, simply called power gain) is defined as a ratio of the power delivered to the load, P_L, to the power input to the block, P_{IB}, that is,

$$G_P = \frac{P_L}{P_{IB}} = \frac{1}{1-|\Gamma_{in}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$



o Available power gain

An available power gain is defined as a ratio of the power available from the block, P_{AVB}, to the power available from the source, P_{AVS}, that is,

$$G_A = \frac{P_{AVB}}{P_{AVS}} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2 \frac{1}{1-|\Gamma_{out}|^2}$$

o Power gain and voltage gain

In RF/RFIC design

- **Power gain is emphasized,**
- **Voltage gain does not mean any sense until the corresponding impedance is specified.**

The transformation between G_P and G_V can be conducted in terms of

$$G_P = \frac{\frac{V_{out}^2}{Z_{out}}}{\frac{V_{in}^2}{Z_{in}}} = G_v^2 \frac{Z_{in}}{Z_{out}}$$

3. Sensitivity of a receiver

o Standard noise source

Noise input power, KTB, is produced by a noisy resistor as a thermal noise :

$$\overline{v^2} = 4kT\Delta fR_n$$

Maximum available noise power from R_N is

$$N_i = \frac{\overline{v^2}}{4R_n} = kT\Delta f$$

As a standard and practical input noise source, kTΔ fis a white noise.

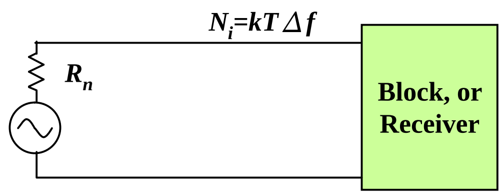


Figure 10.7 Noise source and noise power in the testing of a receiver or a block

It depends on bandwidth but not a given frequency so that it is a white noise.

o Equivalent input noise

We now define another parameter, N_{eq}, the equivalent input noise power of the output noise power,

$$N_{eq} = \frac{N_o}{G}$$

$$NF = \frac{N_o}{GN_i} = \frac{N_{eq}}{N_i} = \frac{N_{eq}}{kT\Delta f}$$

The noise figure, NF, now becomes the ratio of two input noise powers, N_{eq} and N_i. The value of N_{eq} is generally great than $N_i = kT\Delta f$ because the additional noise power of the receiver, ΔN, is added to the output noise power, N_o,

o Sensitivity of 12 dB SINAD

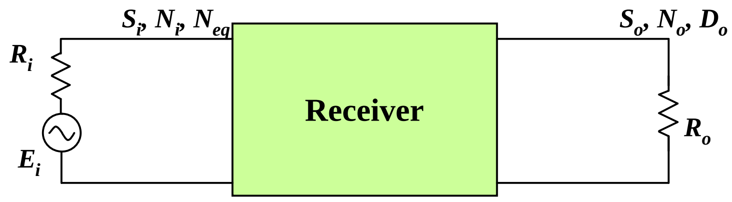


Figure 10.8 The basic parameters in the sensitivity testing for a receiver

Define

$$SINAD = \frac{S_o}{N_o + D_o}$$

Define

$$RISE = \frac{S_i + N_{eq}}{N_{eq}}$$

where

$$S_i = \frac{E_i^{\,2}}{4R_i}$$

Then ,

$$\boxed{E_i = 2\sqrt{(NF)kT\Delta f\,(RISE - 1)R_i}}$$

where

S_i, S_o = Signal power at input and output respectively;
N_i,N_o = Noise power at input and output respectively;
D_o = Distortion power at output;
R_i = Output resistance of generator ;
N_{eq} = Equivalent input noise of output noise;
E_i = Sensitivity looking from input, v

4.Non-linearity and spurious products

o Spurious products

* Spurious are caused by the non-linearity of the device or system
* Spurious are harmonics when input is a signal with only one frequency

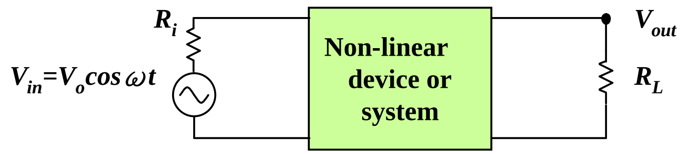


Figure 10.9 The spurious products are produced from the non-linearity in the device or system when the input is a signal with only one frequency.

$$
\begin{aligned}
V_{out} = {} & V_{dc} + a_1V_{in} + a_2V_{in}^2 + a_3V_{in}^3 + a_4V_{in}^4 + \ldots\ldots \\
= {} & V_{dc} + a_1V_o\cos\omega t + a_2V_o^2\cos^2\omega t + a_3V_o^3\cos^3\omega t + a_4V_o^4\cos^4\omega t + \ldots\ldots \\
= {} & V_{dc} + \\
& + a_1V_o\cos\omega t + \\
& + a_2V_o^2[1/2 + (1/2)\cos 2\omega t] + \\
& + a_3V_o^3[(3/4)\cos\omega t + (1/4)\cos 3\omega t] + \\
& + a_4V_o^4[3/8 + (1/2)\cos 2\omega t + (1/8)\cos 4\omega t] + \\
& + \ldots\ldots \\
= {} & [V_{dc} + (1/2)a_2V_o^2 + (3/8)a_4V_o^4 + \ldots] \\
& + [a_1V_o + (3/4)a_3V_o^3 + \ldots\ldots\ldots\ldots]\cos\omega t + \\
& + [(1/2)a_2V_o^2 + (1/2)a_4V_o^4 + \ldots\ldots\ldots]\cos 2\omega t + \\
& + [(1/4)a_3V_o^3 + \ldots\ldots\ldots\ldots\ldots\ldots]\cos 3\omega t + \\
& + [(1/8)a_4V_o^4 + \ldots\ldots\ldots\ldots\ldots\ldots]\cos 4\omega t + \ldots\ldots
\end{aligned}
$$

* Spurious spectrum of one tone

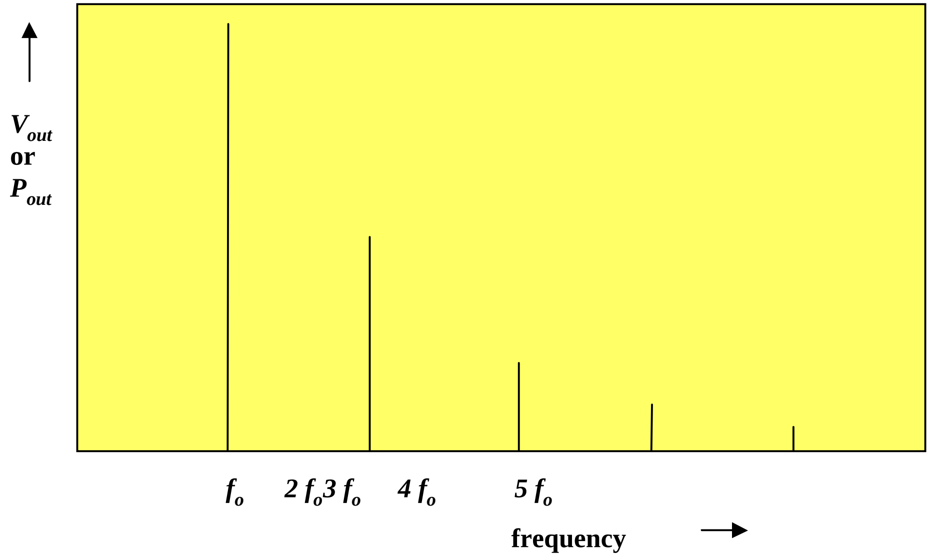


Figure 10.10 The frequency spectrum with harmonics when the input is a pure single frequency.

* Spurious are complicated when input is a signal with two frequencies

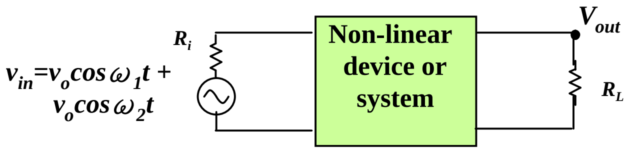


Figure 10.11 Two interference signals produce spurious due to the non-linearity of a device or a system

$$v_{out} = V_{dc} + a_1 v_{in} + a_2 v_{in}^2 + a_3 v_{in}^3 + a_4 v_{in}^4 + \ldots\ldots$$

$$= V_{dc} + a_1 v_o\,[\cos\omega_1 t + \cos\omega_2 t] + a_2 v_o^2\,[\cos\omega_1 t + \cos\omega_2 t]^2 + a_3 v_o^3\,[\cos\omega_1 t + \cos\omega_2 t]^3 + a_4 v_o^4\,[\cos\omega_1 t + \cos\omega_2 t]^4 + \ldots\ldots$$

$$\begin{aligned}
= {} & V_{dc} + a_1 v_o\,[\cos\omega_1 t + \cos\omega_2 t] + \\
& + a_2 v_o^2[1 + (1/2)\cos 2\omega_1 t + (1/2)\cos 2\omega_2 t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t] + \\
& + a_3 v_o^3[(9/4)\cos\omega_1 t + (9/4)\cos\omega_2 t + (1/4)\cos 3\omega_1 t + (1/4)\cos 3\omega_2 t + \\
& \quad +(3/4)\cos(2\omega_1+\omega_2)t + (3/4)\cos(2\omega_2+\omega_1)t + (3/4)\cos(2\omega_1-\omega_2)t + (3/4)\cos(2\omega_2-\omega_1)t] + \\
& + a_4 v_o^4[9/4 + 2\cos 2\omega_1 t + 2\cos 2\omega_2 t + (1/8)\cos 4\omega_1 t + (1/8)\cos 4\omega_2 t + 3\cos(\omega_1+\omega_2)t + 3\cos(\omega_1-\omega_2)t + \\
& (1/2)\cos(3\omega_1+\omega_2)t + \\
& \quad +(1/2)\cos(\omega_1+3\omega_2)t + (1/2)\cos(3\omega_1-\omega_2)t + (1/2)\cos(3\omega_2-\omega_1)t + (3/4)\cos 2(\omega_1+\omega_2)t + (3/4)\cos 2(\omega_1-\omega_2)t] \\
& + \ldots\ldots
\end{aligned}$$

$$\begin{aligned}
= {} & [V_{dc} + a_2 v_o^2 + (9/4)\,a_4 v_o^4] + \\
& + [a_1 v_o + (9/4)\,a_3 v_o^3 + \ldots\ldots]\cos\omega_1 t && + [a_1 v_o + (9/4)\,a_3 v_o^3 + \ldots\ldots]\cos\omega_2 t && + \\
& + [(1/2)\,a_2 v_o^2 + 2a_4 v_o^4 + \ldots]\cos 2\omega_1 t && + [(1/2)\,a_2 v_o^2 + 2a_4 v_o^4 + \ldots]\cos 2\omega_2 t && + \\
& + [a_2 v_o^2 + 3a_4 v_o^4 + \ldots\ldots]\cos(\omega_1-\omega_2)t && + [a_2 v_o^2 + 3a_4 v_o^4 + \ldots\ldots]\cos(\omega_1+\omega_2)t && + \\
& + [a_3 v_o^3 + \ldots\ldots]\cos 3\omega_1 t && + [a_3 v_o^3 + \ldots\ldots]\cos 3\omega_2 t && + \\
& + [(3/4)\,a_3 v_o^3 + \ldots\ldots]\cos(2\omega_2-\omega_1)t && + [(3/4)\,a_3 v_o^3 + \ldots\ldots]\cos(2\omega_2+\omega_1)t && + \\
& + [(3/4)\,a_3 v_o^3 + \ldots\ldots]\cos(2\omega_1-\omega_2)t && + [(3/4)\,a_3 v_o^3 + \ldots\ldots]\cos(2\omega_1+\omega_2)t && + \\
& + [(1/8)\,a_4 v_o^4 + \ldots\ldots]\cos 4\omega_1 t && + [(1/8)\,a_4 v_o^4 + \ldots\ldots]\cos 4\omega_1 t && + \\
& + [(1/2)\,a_4 v_o^4 + \ldots\ldots]\cos(3\omega_2-\omega_1)t && + [(1/2)a_4 v_o^4 + \ldots\ldots]\cos(\omega_1+3\omega_2)t && + \\
& + [(1/2)\,a_4 v_o^4 + \ldots\ldots]\cos(3\omega_1-\omega_2)t && + [(1/2)\,a_4 v_o^4 + \ldots\ldots]\cos(3\omega_1+\omega_2)t && + \\
& + [(3/4)\,a_4 v_o^4 + \ldots\ldots]\cos 2(\omega_1-\omega_2)t && + [(3/4)\,a_4 v_o^4 + \ldots\ldots]\cos 2(\omega_1+\omega_2)t && + \\
& + \ldots\ldots
\end{aligned}$$

* Spurious spectrum of two tones

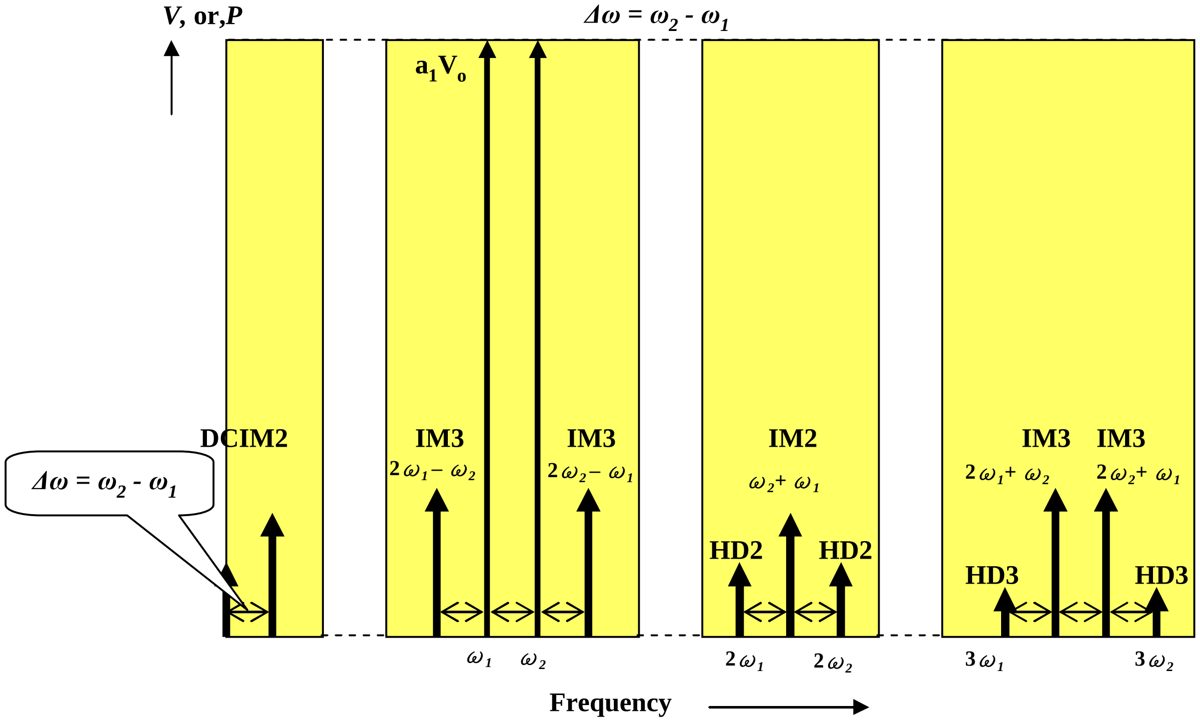


Figure 10.12 The frequency spectrum with harmonics when input is a signal with two frequencies.

* Spurious frequencies of two tones

$$f_{spur.} = |mf_1 - nf_2|$$

where m, n = Zero or positive integer

o Intercept point and IMR (Inter-Modulation Rejection)

* Intercept points

Mthorder intercept point

$$IIP_m = \Delta / (m-1) + P_i$$

$$OIP_m = G + IIP_m$$

where
IIP_m = Mthorder input intercept point
OIP_m = Mthorder output intercept point

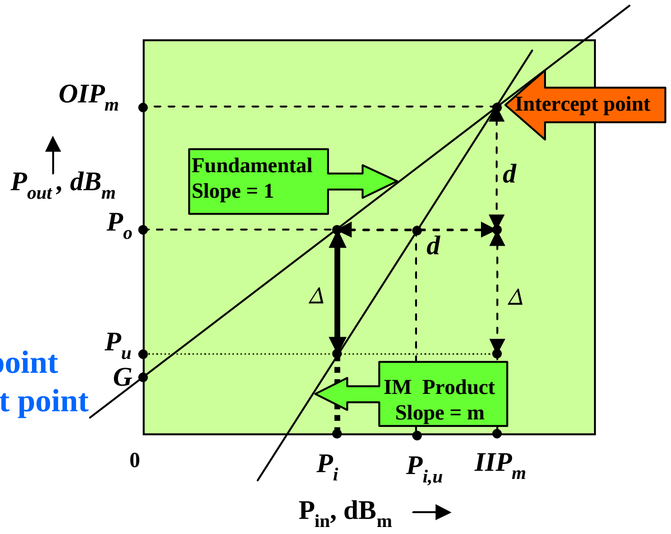


Figure 10.13 Plot of 1st order signal and mthorder IM product on the input-output power plane.

P_i = Input power of signal by dB,
P_o = Output power of signal by dB,
P_u = Undesired output power due to $m$thnon-linearity by dB,
G = Power gain of the device, or block, or system by dB,
OIP_m = The $m$thorder output intercept point by dB,
IIP_m = The $m$thorder input intercept point by dB,
d = Power difference between OIP_m and P_o by dB,
Δ = Output power difference between the 1st order and the $m$thorder by dB.

* IMRm(mthorder Inter-Modulation Rejection)

$$IMR_m = \alpha\ [IIP_m - P_{i,S}]$$

where

α = (m-1)/m

$P_{i,S}$ = Sensitivity (by Power)

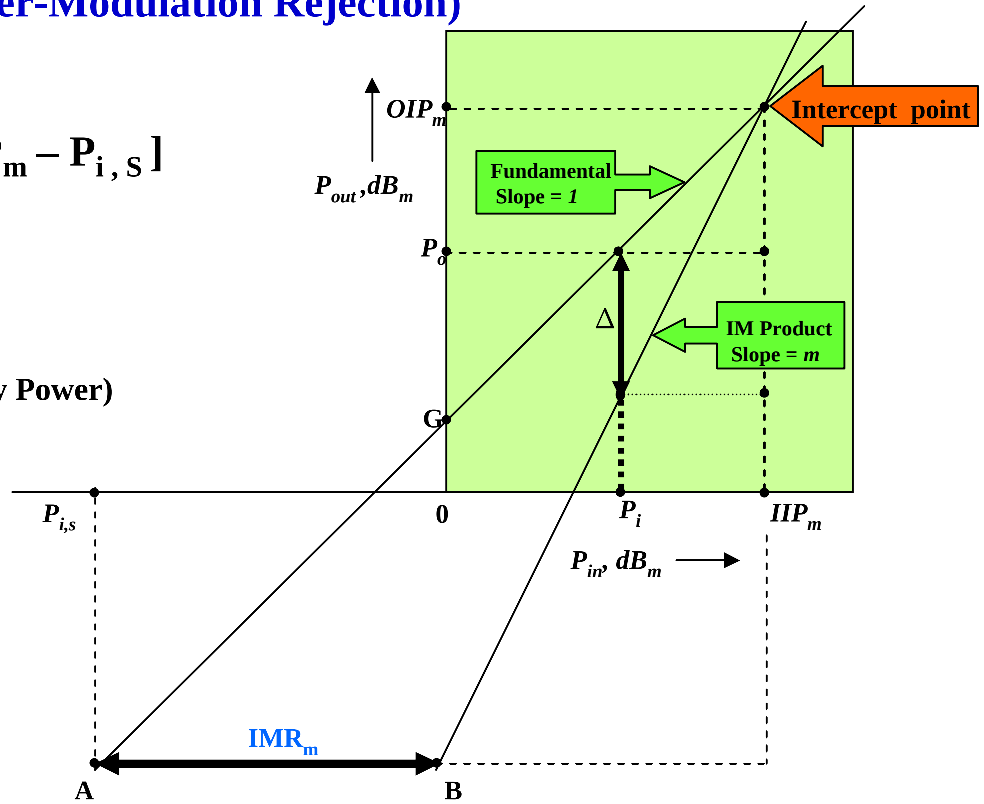


Figure 10.14 Calculation of IMR_m from inteceptpoint, IIP_m.

When $P_{in} = P_{i,S}$, and $P_{in,3rd} = P_{i,S} + IMR_m$, the P_{out} is the same. It implies that This system has a capability to suppress or to reject the $P_{in,3rd}$ down (IMR_m)dB.

We have

<u>For 2nd order spurious</u>,

$$m = 2\,,$$
$$\alpha = 1/2\,,$$
$$IIP_2 = \Delta + P_i\,,$$
$$IMR_2 = (1/2)\,[IIP_2 - P_{i,S}]\,.$$

$$\text{In general, } IMR_2 \approx 20 \log [|2a_1/a_2|^{1/2} / V_o^{-1/2}]$$

<u>For 3rd order spurious</u>,

$$m = 3\,,$$
$$\alpha = 2/3\,,$$
$$IIP_3 = (2/3)\,(\Delta + P_i)\,,$$
$$IMR_3 = (2/3)\,[IIP_3 - P_{i,S}]\,.$$

$$\text{In general, } IMR_3 \approx 20 \log [(4/3)|a_1/a_3|^{1/3}\, V_o^{-2/3}]$$

o The 3^{rd} order spurious

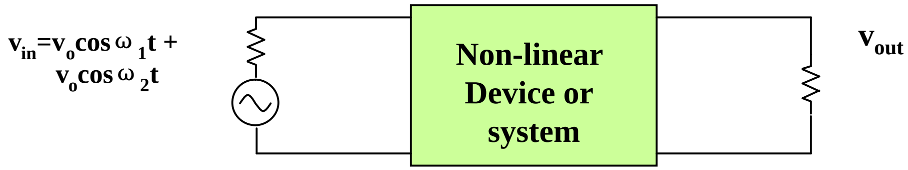


Figure 10.15 Two interference signals produce 3^{rd} order spurious due to the non-linearity of a device or a system

If input is a signal with 2 tones, and assuming that

$$\omega_1 = \omega_d - \Delta\omega, \quad \text{or} \quad f_1 = f_d - \Delta f$$

$$\omega_2 = \omega_d - 2\Delta\omega, \quad \text{or} \quad f_2 = f_d - 2\Delta f$$

the 3^{rd} order non-linear term in the non-linear expression

$$v_{out} = \ldots\ldots + a_3 v_o^3[\ldots\ldots + (3/4)\cos(2\omega_1 - \omega_2)t + \ldots\ldots] + \ldots\ldots$$

$$= \ldots\ldots + a_3 v_o^3[\ldots\ldots + (3/4)\cos\omega_d t + \ldots\ldots] + \ldots\ldots$$

Produces a spurious product at the frequency ω_d, that is

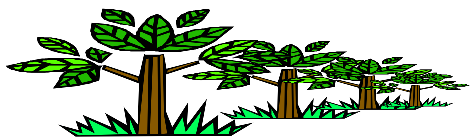
$$\omega = 2\omega_1 - \omega_2 = \omega_d$$

The specialityis that

Δ f could be in any value !

It implies that

In a non-linear system the 3^{rd} order spurious can never be eliminated by means of any filtering technology.



Where
Signal with frequency, ω_1, or, ω_2 = Undesired signal, while
Signal with frequency, ω_d = desired signal .

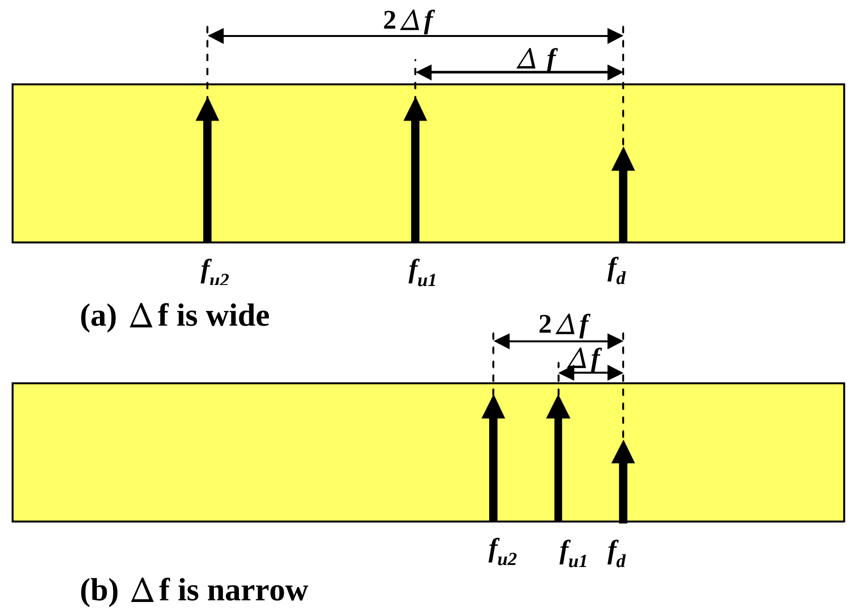


Figure 10.16 **Undesired signal frequencies, f_{u1} and f_{u2}, and desired signal frequency, f_d.**

Setup for IP3 testing

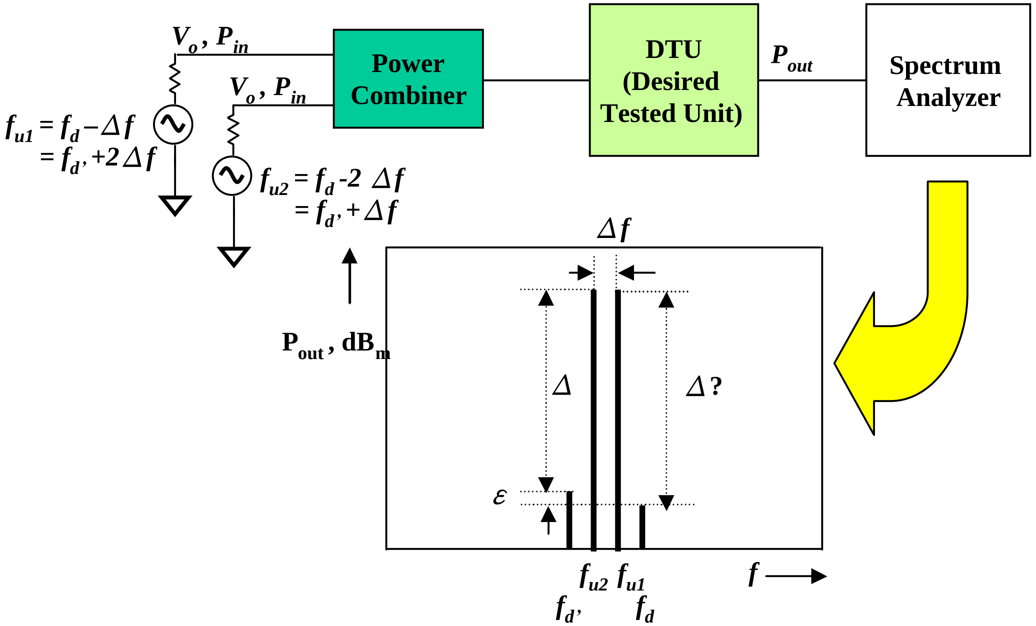


Figure 10.17 Set-up and display of ***IP_3*** **testing**

The value of Δ in both of left and right side could be different. One should read the One with lower value, which corresponds to the orstcase.

o 1 dB compression and IIP_3

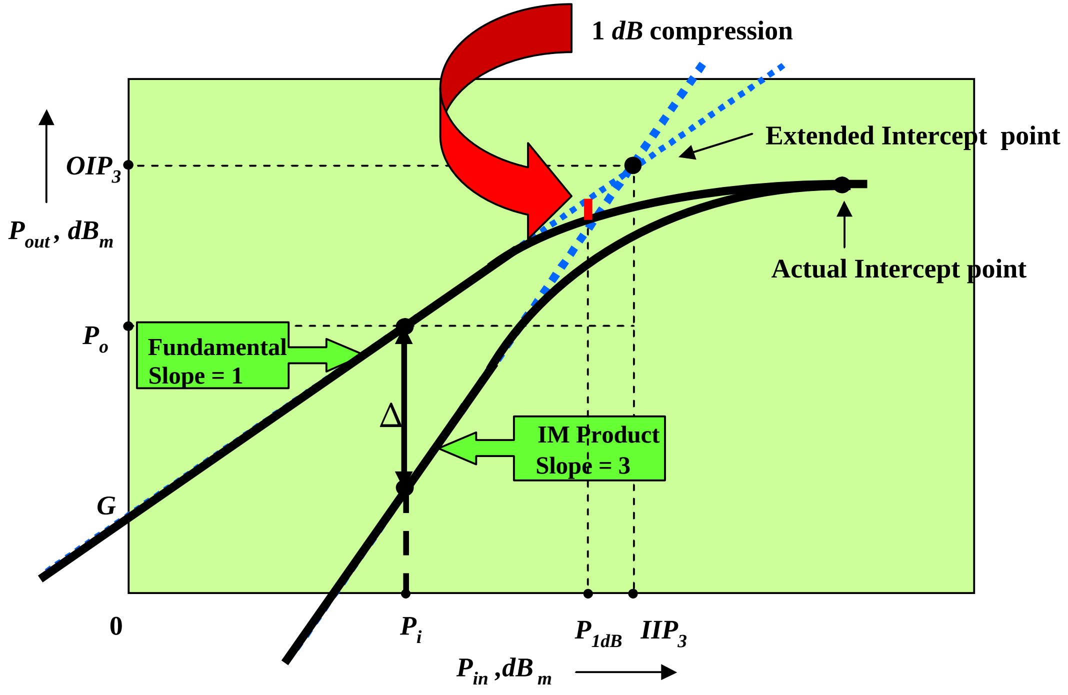


Figure 10.18 1 *dB* compression point and IP_3

$$IIP_3 \text{ à } P_{1dB} + (3 \text{ to } 10)\ \text{dB}$$

For intercept point testing, usually keep

$$P_i < -30\ \text{dBm}$$

o The 2^{nd} order intercept point

- * It is a spurious that mostly closed to the IF frequency,
- * It is a representative in the measurement of DC offset for asystem,
- * Theoretically the differential configuration has zero DC offset,
- * Testing of 2^{nd} order intercept point.

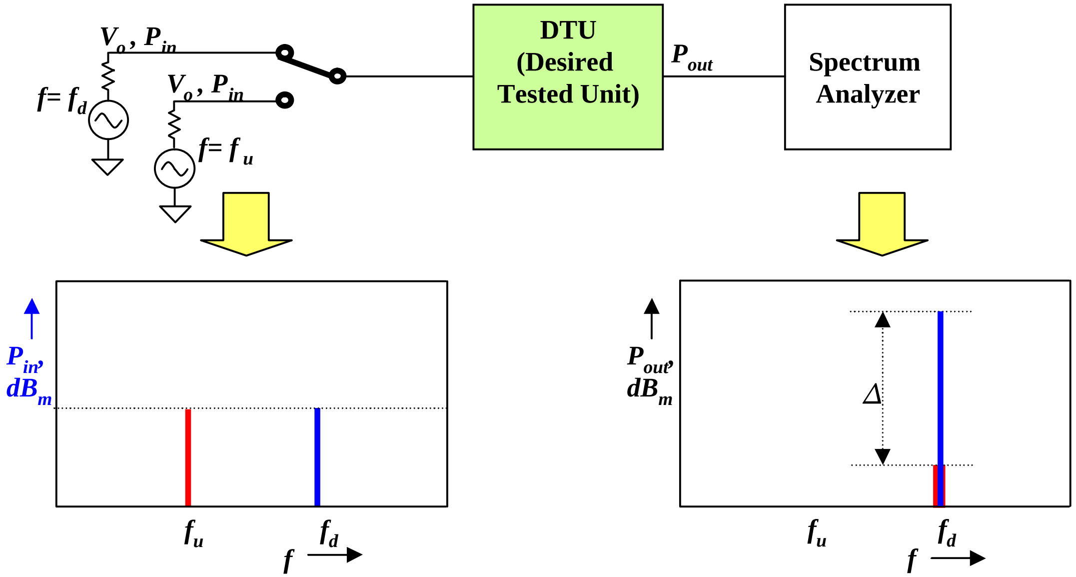


Figure 10.19Set-up and display of IP_2 testing

5. Distortion

- Distortion is formed by the sum of all the spurious and harmonics
- Percentage of distortion and dB of distortion

$$D(dB) = 20\log[D(\%)]$$

Example : 5% of distortion is equal to about –26 dB

$$\boxed{(-26.02)\,dB = 20\log(\frac{5}{100})}$$

* Power Amplifier

- **PAE (Power Added Efficiency)**

$$\mathbf{PAE = (P_{out} - P_{in})/P_{dc}}$$

where P_{out} = **Output RF power;**
P_{in} = **Input RF power;**
P_{dc}= **Power consumption of DC power supply.**

- **Directional couplers must be applied in testing**

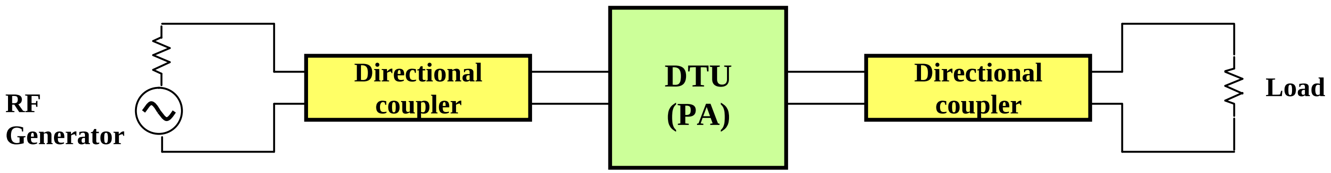


Figure 10.20 Set-up for PAtesting

6. Cascaded equations

- **Gain (dB)**

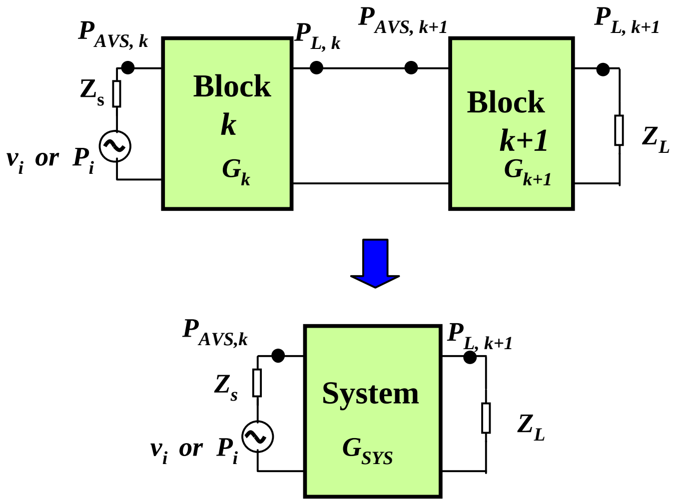


Figure 10.21 The system gain cascaded by two gains of block *k* and *k+1*.

$$G_{SYS} = G_i + G_{i+1}$$

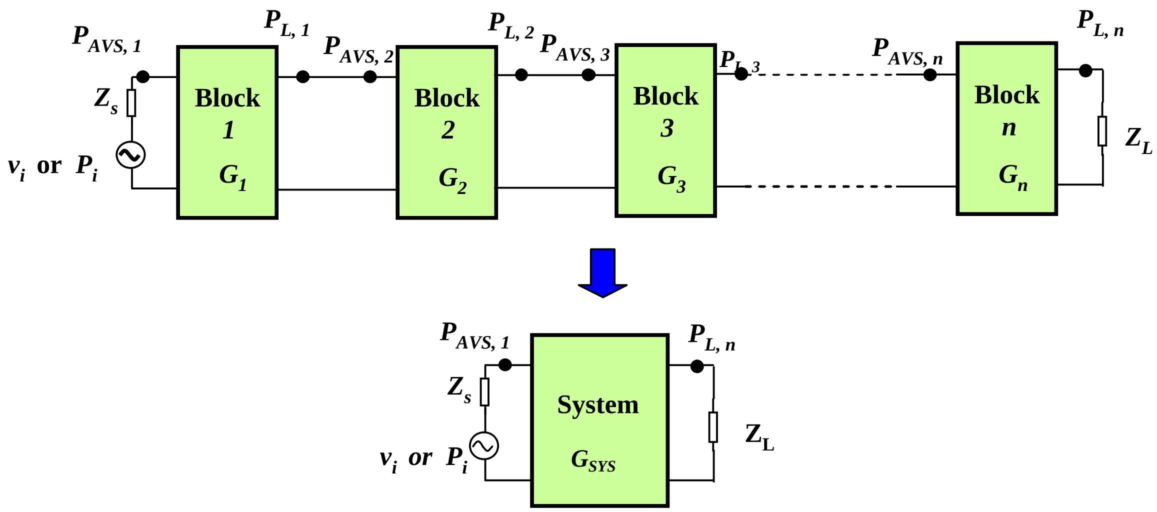


Figure 10.22The system gain cascaded by individual gains from block *1* to block *n*.

$$G_{SYS,dB} = G_1 + G_2 + G_3 + \ldots + G_n$$

$$G_{SYS,dB} = \sum_{1}^{n} G_k$$

o Noise Figure

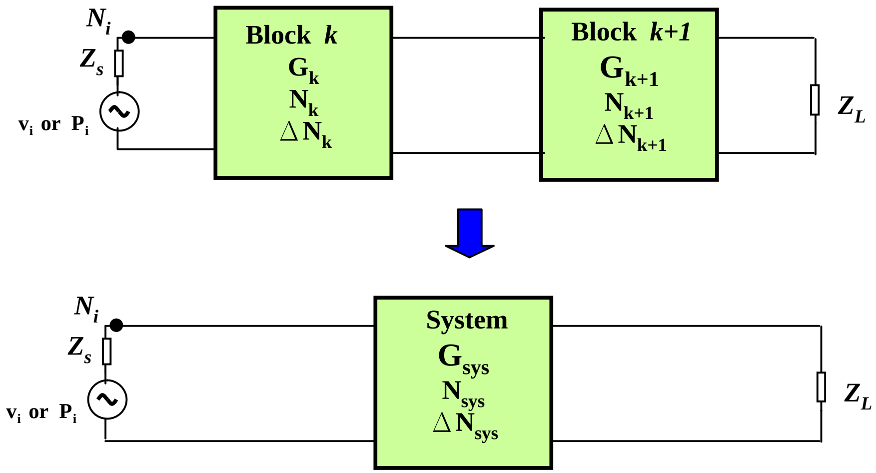
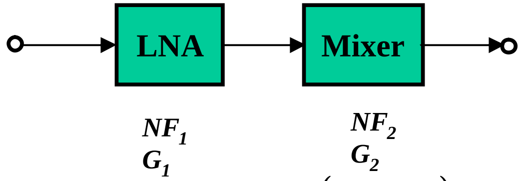


Figure 10.23The system noise figure cascaded by two noise figures of block *k* and *k+1*.

(Power) $NF_{SYS,watt} = NF_i + (NF_{i+1} - 1)/G_i$

o Approach to the identical system noise figure by different LNA and FLT



$$NF_{SYS,watt} = NF_1 + \frac{(NF_2 - 1)}{G_1}$$

Figure 10.24System noise figure if a system is cascaded by block 1 and 2.

$$NF_{SYS\ ,watt} = NF_1 + \frac{(NF_2 - 1)}{G_1}$$

Tabal10.1 : The same system noise figure can be obtained by different combinations of noise figures from individual blocks

Example #1	Block 1 LNA	Block 2 FLT	System
NF, dB	2	3	2.17
Gain, dB	12	-3	9.00

Example #2	Block 1 LNA	Block 2 FLT	System
NF, dB	2	7	2.17
Gain, dB	18	-7	11.00

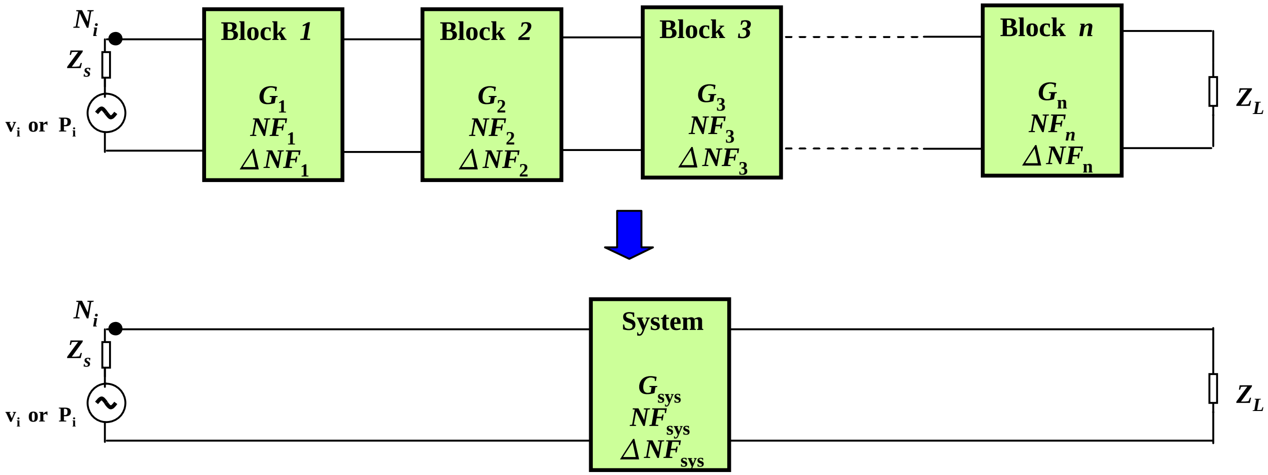


Figure 10.25The system noise figure cascaded by individual noise figures from block *1* to block *n*.

$$NF_{SYS,watt} = NF_1 + \frac{(NF_2 - 1)}{G_1} + \frac{(NF_3 - 1)}{G_1 G_2} + \frac{(NF_4 - 1)}{G_1 G_2 G_3} + \ldots + \frac{(NF_n - 1)}{G_1 G_2 \ldots G_{n-1}}$$

$$NF_{SYS,watt} = NF_1 + \sum_{2}^{n} \frac{(NF_k - 1)}{\prod_{1}^{k-1} G_j}$$

o m^{th} Intercept point

$IIP_{m,\ SYS}^{-(m-1)/2} = IIP_{m,\ i}^{-(m-1)/2} + [IIP_{m,\ i+1}/G_{i}]^{-(m-1)/2}$ **(Power)**

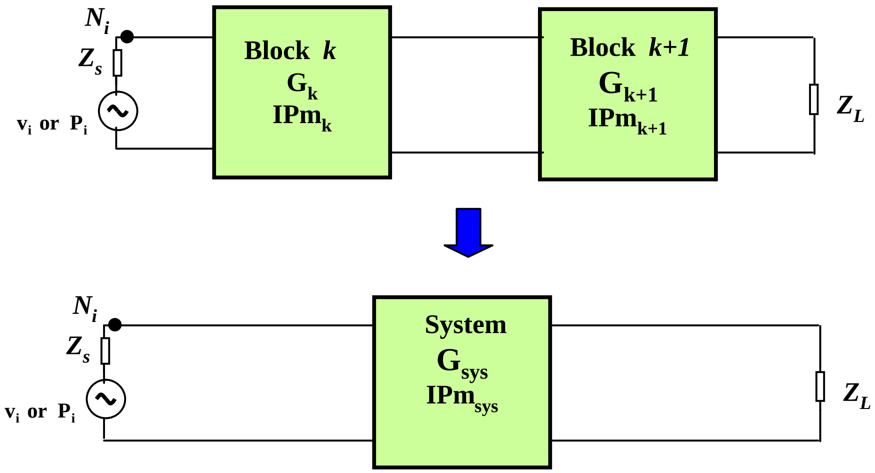


Figure 10.26The system noise figure cascaded by two noise figures of block *k* and *k+1*.

The intercept point with *m*thorder :

- P_i = Input power of signal by *dB*,
- P_o = Output power of signal by *dB*,
- P_u = Undesired output power due to *m*thnon-linearity by *dB*,
- G = Power gain of the device, or block, or system by *dB*,
- OIP_m = The *m*thorder output intercept point by *dB*,
- IIP_m = The *m*thorder input intercept point by *dB*,
- d = Power difference between *OIPm* and *Po* by *dB*,
- Δ = Output power difference between the 1st order and the *m*thorder by *dB.*

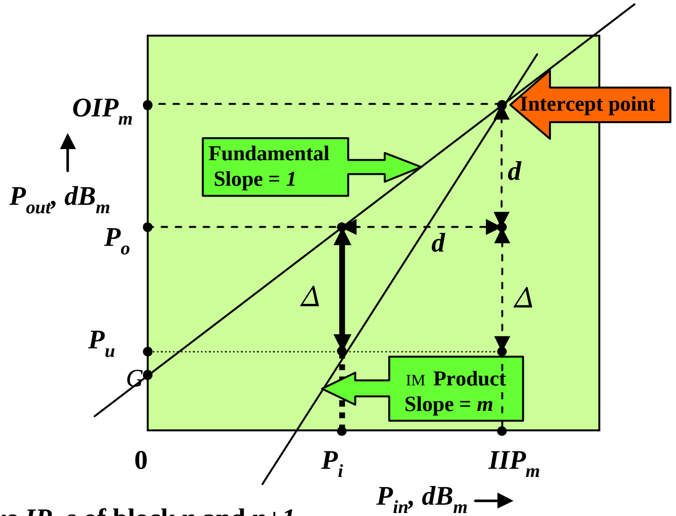


Figure 10.27The system IP_m cascaded by two IP_ms of block *n* and *n+1*.

o Cascade IIP3 when G1 or IIP2 is varied

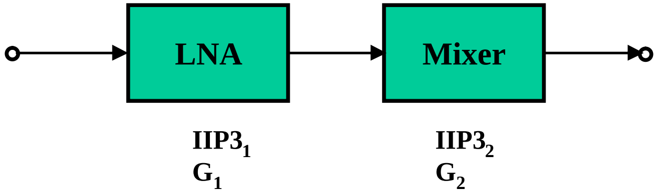


Figure 10.28The system intercept point cascaded by individual intercept points of two block *s*

$$\left(\frac{1}{IIP_{sys,watt}}\right) = \left(\frac{1}{IIP3_1}\right) + \left(\frac{G_1}{IIP3_2}\right)$$

Tabal10.2 System IIP3 is calculated from the IIP3s of individual blocks

	Block 1:LNA	Block 2 :Mixer	System
Example #2			
IIP3, dBm	**2**	**10**	**-3.5**
Gain, dB	**12**	**0**	**12.0**
Example #2			
IIP3, dBm	**100**	**10**	**-2.0**
Gain, dB	**12**	**0**	**12.0**
Example #3			
IIP3, dBm	**2**	**100**	**2.0**
Gain, dB	**12**	**0**	**12.0**
Example #4			
IIP3, dBm	**2**	**2**	**-1.0**
Gain, dB	**0**	**0**	**0**

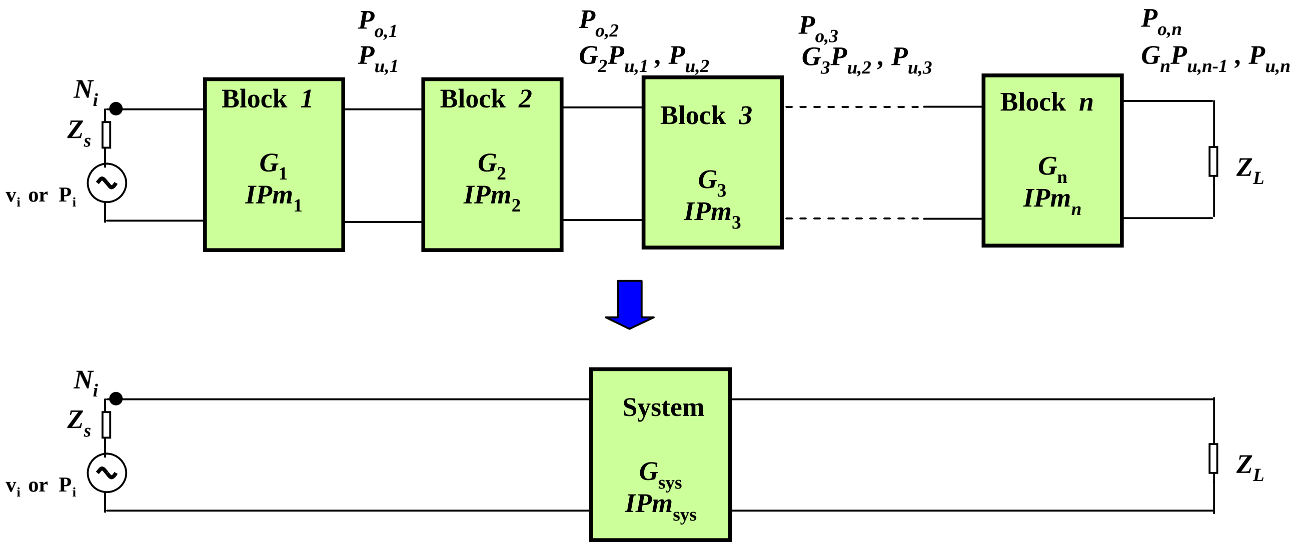
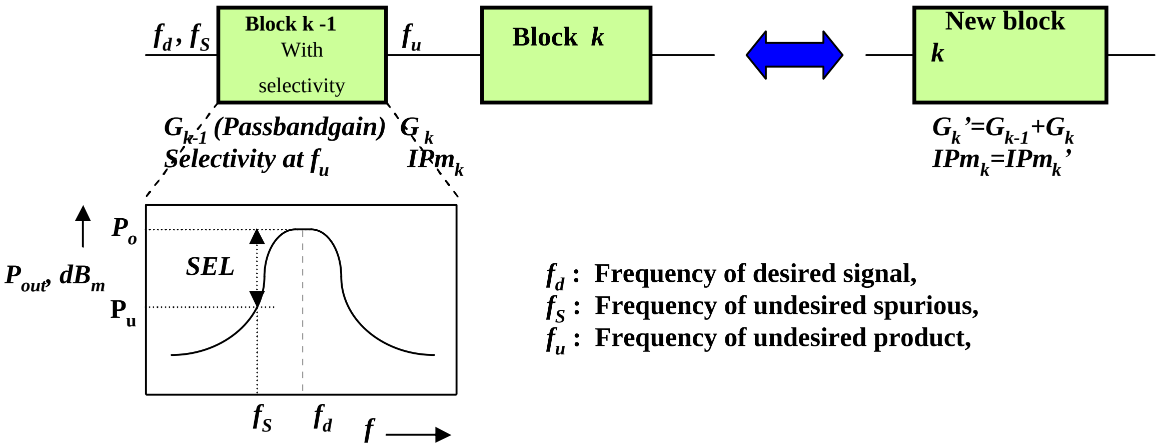


Figure 10.29The system intercept point cascaded by individual intercept points from block *1* to block *n*.

$$\left(\frac{1}{IIP_{m,T}}\right)^{\frac{(m-1)}{2}} = \left(\frac{1}{IIP_{m,1}}\right)^{\frac{(m-1)}{2}} + \left(\frac{G_1}{IIP_{m,2}}\right)^{\frac{(m-1)}{2}} + \left(\frac{G_1G_2}{IIP_{m,3}}\right)^{\frac{(m-1)}{2}} + \Lambda + \left(\frac{G_1G_2G_3\Lambda G_{n-1}}{IIP_{m,n}}\right)^{\frac{(m-1)}{2}}$$

$$\left(\frac{1}{IIP_{m,T}}\right)^{\frac{(m-1)}{2}} = \left(\frac{1}{IIP_{m,1}}\right)^{\frac{(m-1)}{2}} + \sum_{2}^{n}\left(\frac{\prod_{1}^{k-1} G_j}{IIP_{m,k}}\right)^{\frac{(m-1)}{2}}$$

o IP_m cascaded equation when the selectivity is existed in the block



f_d : Frequency of desired signal,
f_S : Frequency of undesired spurious,
f_u : Frequency of undesired product,

Figure 10.30 Combination of two blocks into one block when the selectivity is existed in the first one .

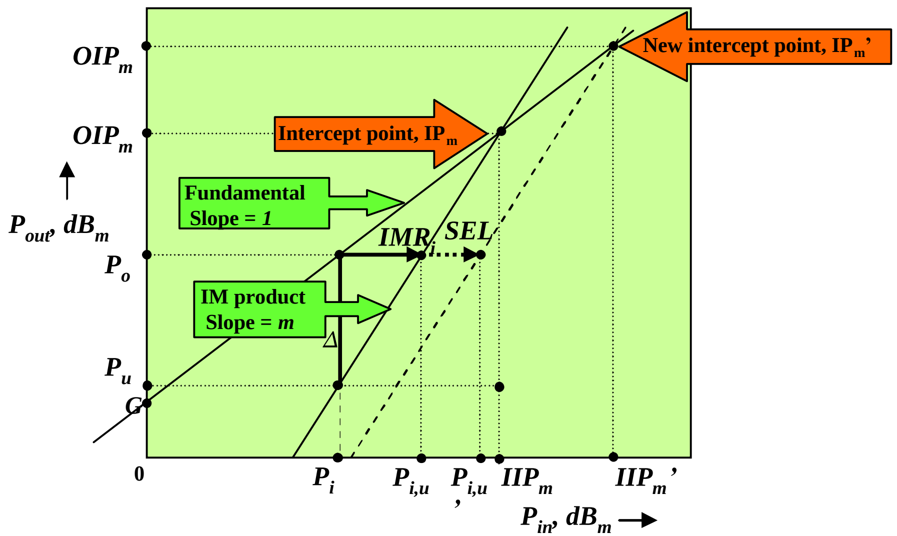


Figure 10.31The relationship between IP_m and IP_m' when $P_{i,u}$ is moved to $P_{i,u}$'.

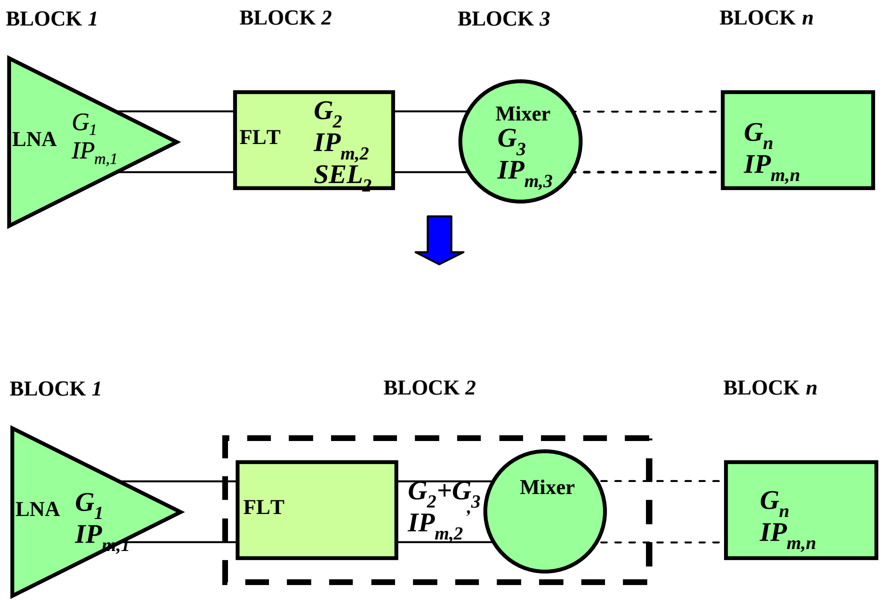


Figure 10.32In the case of the block with selectivity, the block should be combined with the following block to form a new block.

By dB :

$$IIP_m' = IIP_m + \frac{m}{m-1} SEL$$

By watt :

$$IIP_m' = IIP_m SEL^{\frac{m}{m-1}}$$

$$OIP_m' = GIIP_m' = GIIP_m SEL^{\frac{m}{m-1}}$$

o Application of cascaded equation system analysis : Analysis of RX front end

Table 10.3 System analysis of Rx front end

System goals

Lower Freq. :	403 MHz	Temperature	: 298 K°
Upper Freq. :	470 MHz	RISE@12dB SINAD :	6.0 dB
IF Freq. :	73.35 MHz	RISE@20dB Quiet :	7.6 dB
System BW :	13.5 kHz	Worst case Factor :	0.1

Performance	Ant.SW	FLT#1	RF Amp.	FLT#2	Mixer	XtalFLT	IF Amp.	XtalFLT	Bk.End
Gain, dB	-0.9	-2	12	-1.5	-1	-2	10	-2	
NF, dB	0.9	2	2.5	1.5	7.5	2	3	2	6
IP3, dBm	30	18	10	100	18	1000	12	1000	20
IP2, dBm	100	100	100	100	48	1000	100	1000	1000
SEL@Δf, dB	0	0	0	0	0	17	0	17	0
SEL@2Δf, dB	0	0	0	0	0	29	0	29	0
SEL@1/2 IF, dB	0	1 0	0	0	0	0	0	0	0

Calculations	Ant.SW	FLT#1	RF Amp.	FLT#2	Mixer	XtalFLT	IF Amp.	XtalFLT	Bk.End
Gain (worst), dB	-1	-2.2	10.8	-1.7	-1.1	-2.2	9.0	-2.2	
NF (worst), dB	1.0	2.2	2.8	1.7	8.3	2.2	3.3	2.2	6. 6
NFsys, dB	7.0	6.1	4.1	11.2	9.7	6.0	4.0	8.0	6.0
Nfsys(worst) dB	8.3	7.3	5.1	12.3	10.7	6.9	4.7	8.8	6.6
12dBSINAD uv	0.20	0.18	0.15	0.33	0.28	0.18	0.14	0.23	0.18
12dBSINAD dBm	-120.8	-121.7	-123.7	-116.6	-118.1	-121.8	-123.8	-119.8	-121.8
20dBQuiet, uv	0.26	0.23	0.18	0.42	0.35	0.23	0.18	0.29	0.23
20dBQuiet, dBm	-118.8	-119.7	-121.7	-114.6	-116.1	-119.8	-121.8	-117.8	-119.8
IIP3sys, dBm	8.1	7.2	5.6	19.5	18.0	45.5	12.0	53.5	20.0
IMR3 dB	85.9	85.9	86.2	90.7	90.7	111.5	90.5	115.5	94.5
IIP2sys, dBm	52.7	50.9	27.0	51.0	48.0	104.0	100.0	995.8	1000
IMR2 dB	86.8	86.3	75.3	83.8	83.0	112.9	111.9	557.8	560.9

7. From analog to digital communication system

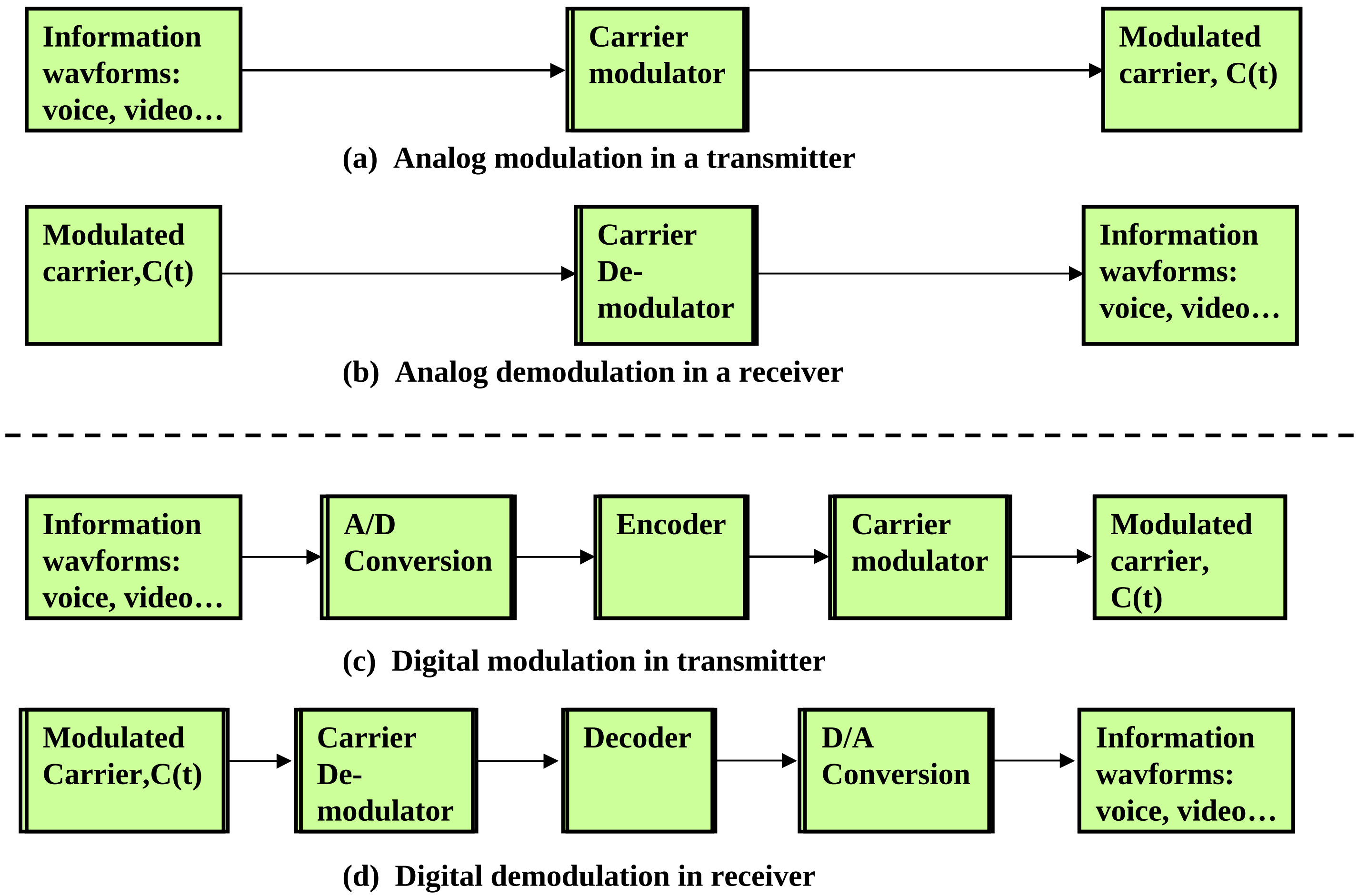


Figure 10.33The basic components in both analog and digital modulation system

- **Modulation in an analog communication system**

For AM modulation,

$$C(t) = A[1 + \Delta_a m(t)]\cos(w_c t + \Psi)$$

$$P_c = \frac{A^2}{2}(1 + \Delta_a^2 P_m)$$

For FM modulation,

$$C(t) = A\cos(w_c t + 2p\Delta_f \int m(t)dt + \Psi)$$

$$P_c = \frac{A^2}{2}$$

For PM modulation,

$$C(t) = A\cos(w_c t + \Delta_p m(t) + \Psi)$$

$$P_c = \frac{A^2}{2}$$

o Modulation in an analog communication system

Table 10.4 Key parameters for three types of modulation

Modulation type	BW_n	Required CNR	Demodulated SNR
AM	$2BW_m$	> 12 dB	=CNR
FM	$2(\beta+1)BW_m$	> 16 dB	$6\beta^2(\beta+1)CNR$
PM	$2(\Delta_p+1)BW_m$	> 10 to 12 dB	$6\Delta_p^2(\Delta p+1)CNR$

Notes:
BW_n = Noise bandwidth ;
BW_m = Bandwidth of m(t);
β = Peak frequency deviation/BW_m;
Δp= Phase deviation.

Table 10.5 Bandwidth of some sources, BW_m.

Type of source	BWm
Voice	4 kHz
Multiplexed voice, 1000 voice sub-channels	4MHz
Video	4 MHz
Television channel	6 MHz

o Encoding in a digital communication system

* **Baseband waveform**

$$m(t) = \sum_{i=-\infty}^{\infty} d_i W(t - iT_b)$$

$$d_i = \pm 1$$

where d_i **= i**th **antipodal digit.**

A digital data bit "1" is encoded into a specific waveform when d_i**=1, while a digital data bit "0" is encoded into the negative of that waveform when** d_i**=-1.**

W **= a specific waveform representing a binary data bit "1",**

t **= time,**

T_b **= bit time.**

The bit rate is the reciprocal of T_b**, that is,**

$$R_b = \frac{1}{T_b}$$, bits/sec .

*** NRZ and Manchester waveforms**

NRZ waveform $W(t) = 1$, **when** $0 \le t \le T_b$

= 0 , **Elsewhere**

Manchester waveform $W(t) = 1$, **when** $0 \le t \le \frac{T_b}{2}$

= -1 , **when** $\frac{T_b}{2} \le t \le T_b$

Spectrum of the baseband waveform, m(t),

$$S_m(w) = \frac{1}{T_b} \left| W(w) \right|^2$$

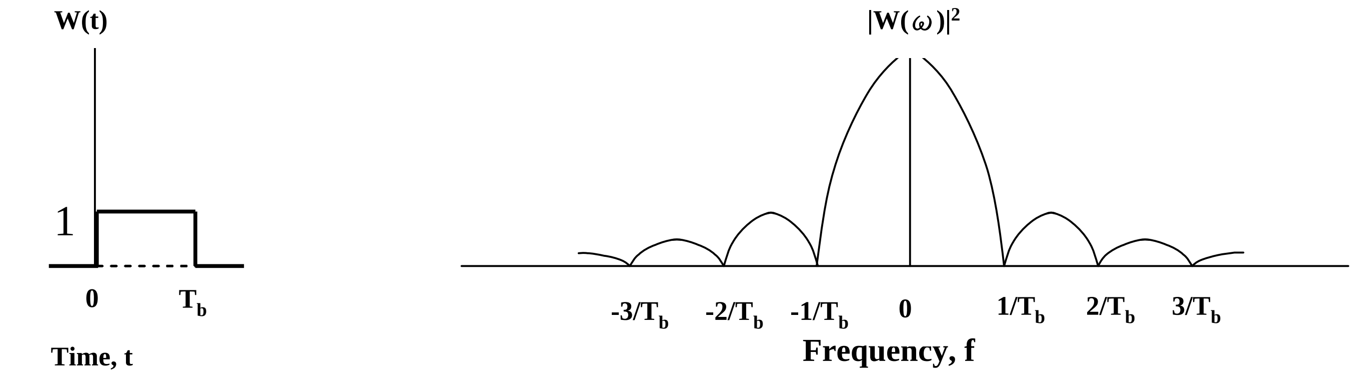
where W(ω) is the Fourier Transform of W(t).

The spectral density for NRZ waveform

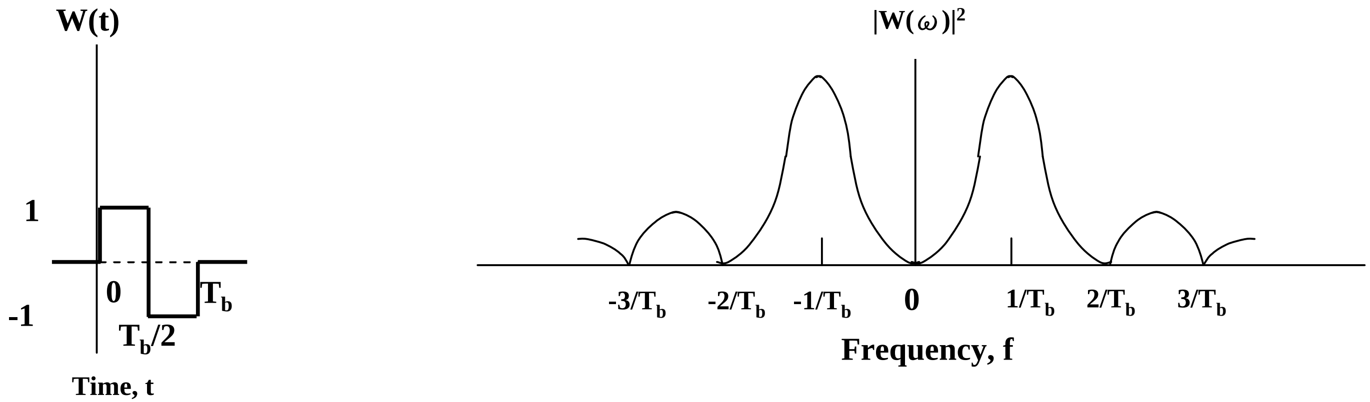
$$\left| W(w) \right|^2 = \left| \frac{\sin\left(pfT_b\right)}{pfT_b} \right|^2$$

The spectral density for Manchester waveform

$$\left| W(w) \right|^2 = \left| \frac{\sin^4\left(pf\,\frac{T_b}{2}\right)}{\left(pf\,\frac{T_b}{2}\right)^2} \right|^2$$



(a) NRZ waveform and spectrum



(b) Manchester waveform and spectrum

Figure 10.34Baseband waveforms and spectra

* **BPSK (Binary Phase Shift Keying)**

The modulated phase, $\Delta_p m(t)$,

$$\Delta_p m(t) = \frac{p}{2}[1 - m(t)] \qquad \begin{array}{ll} =0, & \text{when } m(t) = 1 \\ = \pi, & \text{when } m(t) = -1. \end{array}$$

$$C(t) = A\cos(w_c t + \frac{p}{2}[1 - m(t)] + \Psi)$$

Then, in terms of the triangular formula

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b),$$

because the value of the modulated phase is either 0 or π, we have

$$C(t) = Am(t)\cos(w_c t + \Psi)$$

It means that the BPSK carrier is the same as an amplitude-modulated carrier in which the baseband waveform, *m(t)*, directly multiplies the carrier.

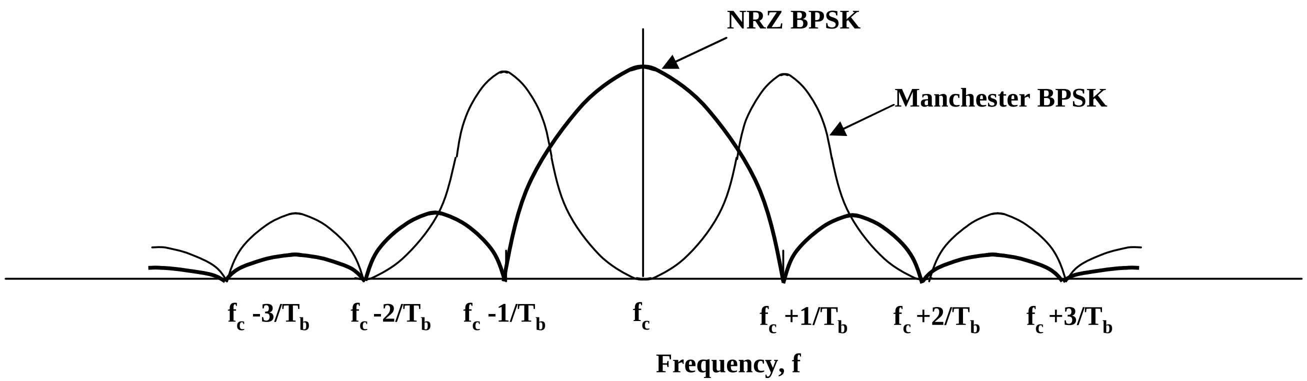


Figure 10.35 BPSK carrier spectra

*   **RC (Raised Cosine) pulse: to reduce the sub-humps or the spectral tails**

$$W(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi t}{T_b}\right)\right] \qquad 0 \le t \le T_b$$

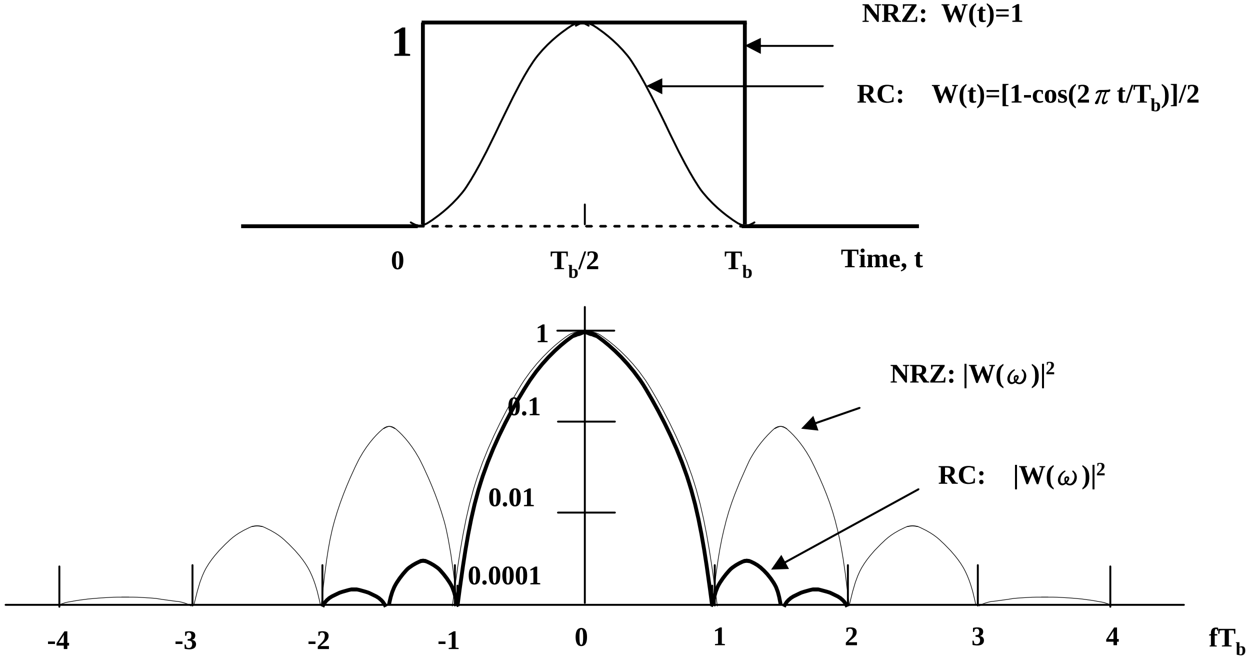
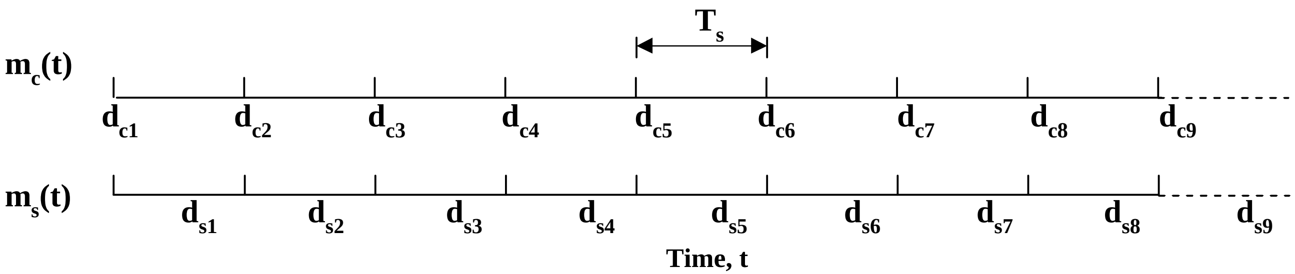


Figure 10.36   NRZ and RC waveform and their spectra

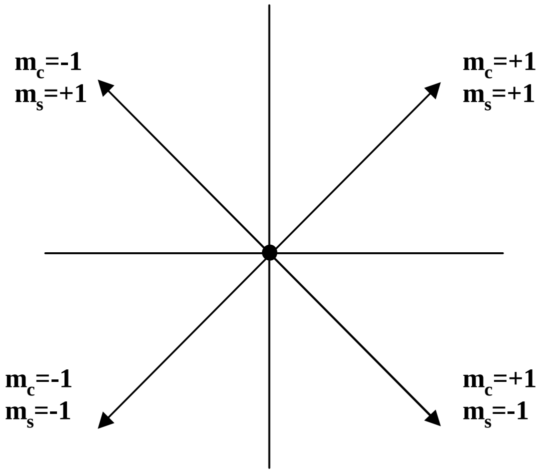
The corresponding spectral density of RC waveform is

$$|W(\omega)|^2 = \left|\frac{\sin(\pi f T_b)}{\pi f T_b}\right|^2 \left|\frac{\cos(\pi f T_b)}{1-(2 f T_b)^2}\right|^2$$

- **QPSK (QuadraturePhase Shift Keying), OQPSK, MSK**
  - **Two encoded waveforms, $m_c(t)$ and $m_s(t)$, are simultaneously used to BPSK phase modulate the same carrier with quadraturephases.**
  - **These two encoded waveforms can be either the output from two different digital sources or the output of the alternate bits from a common source.**



(a) Bit sequence alignment of $m_c(t)$ and $m_s(t)$



$$T_b = \frac{T_s}{2}$$

whereT_b=Bit time,

T_s=Symbol time.

(b) Quadraturephase relationship

Figure 10.37 QPSK encoding

The QPSK carrier can be expressed as

$$C(t) = Am_c(t)\cos(w_c t + \Psi) + Am_s(t)\sin(w_c t + \Psi)$$

Or,

$$C(t) = \sqrt{2} A a(t) \cos(w_c t + q(t) + \Psi)$$

$$\sqrt{2} a(t) = \sqrt{m_c^2(t) + m_s^2(t)}$$

$$q(t) = \tan^{-1}\left[m_s(t) / m_c(t)\right]$$

* **MSK (Minimum Shift Keyed)**

It is a special OQPSK, in which the NRZ waveform is replaced by half-period sine waveform, that is,

$$W(t) = \sin(\frac{p}{T_s})t \qquad 0 \le t \le T_s$$

$$W(t - \frac{1}{2}T_s) = \sin\left[\frac{p}{T_s}(t - \frac{1}{2}T_s)\right] = \cos(\frac{p}{T_s})t \qquad 0 \le t \le T_s$$

The resulting MSK carrier is

$$C(t) = A\sum_{i=-\infty}^{\infty} d_{ci} \sin\left[\frac{p}{T_s}(t - iT_s)\right]\cos(w_c t + \Psi) + A\sum_{i=-\infty}^{\infty} d_{si} \cos\left[\frac{p}{T_s}(t - iT_s)\right]\sin(w_c t + \Psi)$$

$$C(t) = A\cos\left[w_c t + d_i \frac{p}{T_s}(t - iT_s) + c\right] \text{-----------------} \quad iT_s \le t \le (i+1)T_s$$

$$C(t) = A\cos\left[2p\,(f_c + \frac{d_i}{4T_b})t - id_i p + c\right] \text{-----------------} \quad i2T_b \le t \le (i+1)2T_b$$

$$d_i = d_{ci} \quad \text{---} \quad iT_b \le t \le (i+1)T_b$$

$$d_i = d_{si} \quad \text{---} \quad (i+1)T_b \le t \le (i+2)T_b$$

$$c_i = c_{i-1} + \frac{pi}{2}(d_{i-1} - d_i) + \Psi$$

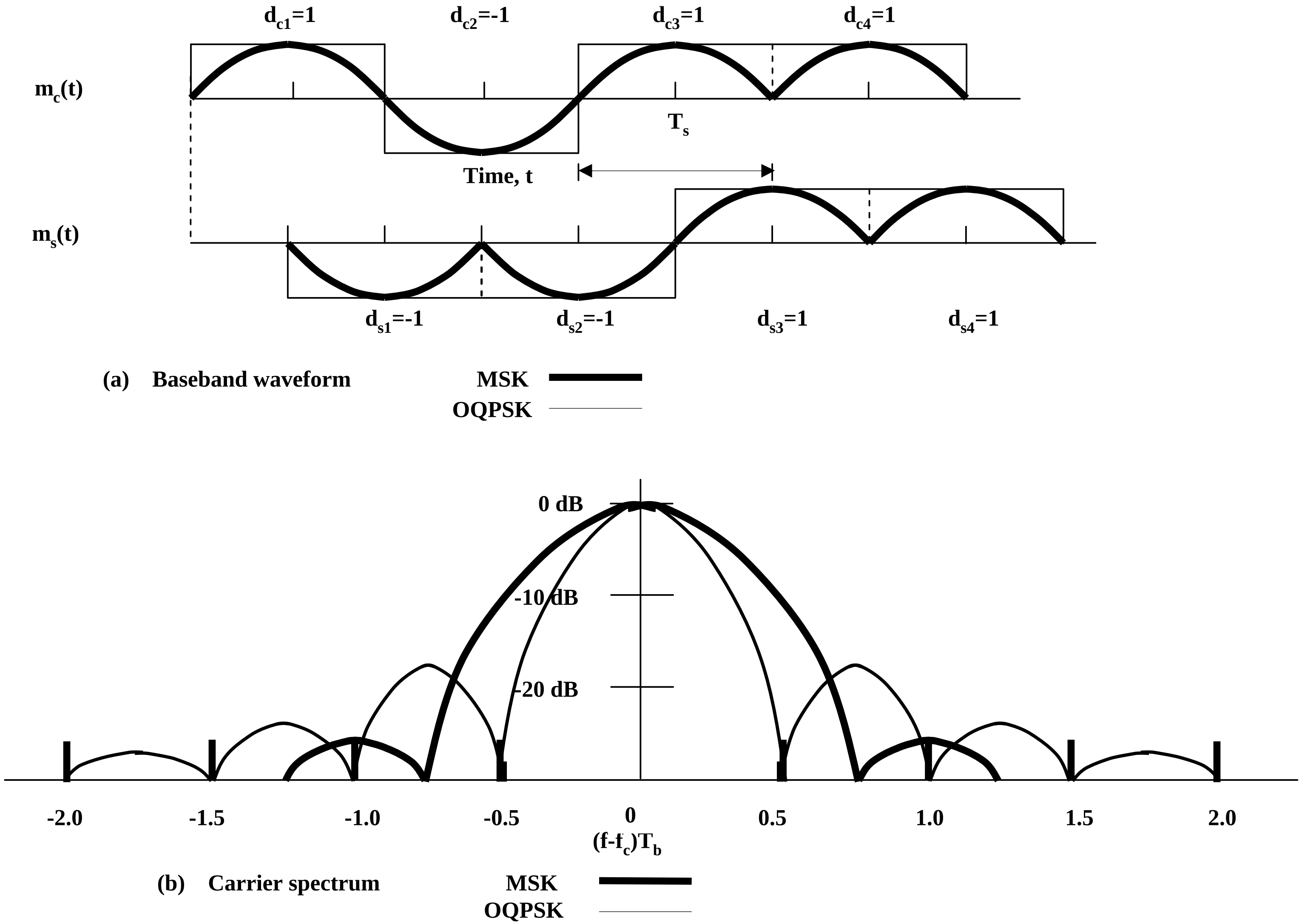


Figure 10.38 MSK and OQPSK baseband waveform and spectrua

Three advantages of MSK carrier :

1) **MSK carrier a constant envelope carrier.**
2) **The spectral tail in MSK modulation is significantly reduced from that of OQPSK modulation.**
3) **It is a carrier with frequency-shifted between (f_c-1/(4T_b)) and (f_c+1/(4T_b)), according to the data bits. The tone spacing in MSK is one-half that employed for non-coherently demodulated orthogonal FSK. It is therefore named as "minimum shift keying."**

* **FSK (Frequency Shift Keying), CPFSK**

In FSK modulation, the NRZ bit waveform modulates the carrier so that the carrier frequency is shifted between two modulation frequencies, according to the bit waveform.

The modulated carrier has constant envelope :

$$C(t) = A\cos\left[w_c t + 2p\Delta_f \int m(t)dt + \Psi\right]$$

It is somewhat difficult to compute the carrier spectrum.
Its main hump occupies from $-0.5(f-f_c)T_b$ to $+0.5(f-f_c)T_b$.

FSK is often referred to as CPFSK (Continuous Phase Frequency Shift Keying) if the NRZ waveform is replaced by the RC waveform. Its frequency–shifted carriers has the property of continuous phase change.

o Decoding and bit-error probability

Table 10.6 BER (Bit Error Rate) in various decoding schemes

BPSK, QPSK, MSK	$Q[(2E_b/N_o)^{0.5}]$
Coherent FSK	$Q[(E_b/N_o)^{0.5}]$
DPSK	$\exp^{(-Eb/No)}/2$
NoncoherentFSK	$\exp^{(-Eb/2No)}/2$

$$Q[x] = \frac{1}{2p} \int_x^{\infty} e^{-t^2/2}\, dt$$

$$\frac{E_b}{N_o} = \frac{Bit\_energy\_of\_the\_modulated\_carrier\_at\_decoder\_inpu}{Spectral\_level\_of\_additive\_noise\_at\_decoder\_input}$$

$$\frac{E_b}{N_o} = \frac{P_c T_b}{N_o}$$

Where P_c **= Power of carrier,**
T_b **= Bit time,**
N_o **= Noise power**

* coherent & non-coherent decoding

coherent decoding requires the decoder to use a referenced carrier at the same frequency and phase as the received modulated carrier during each bit time. A synchronization subsystem is therefore required in the receiver.

On the contrary, the non-coherent decoding does not required a reference carrier, in which the carrier phase of each bit is referenced to the previous bit, that is,

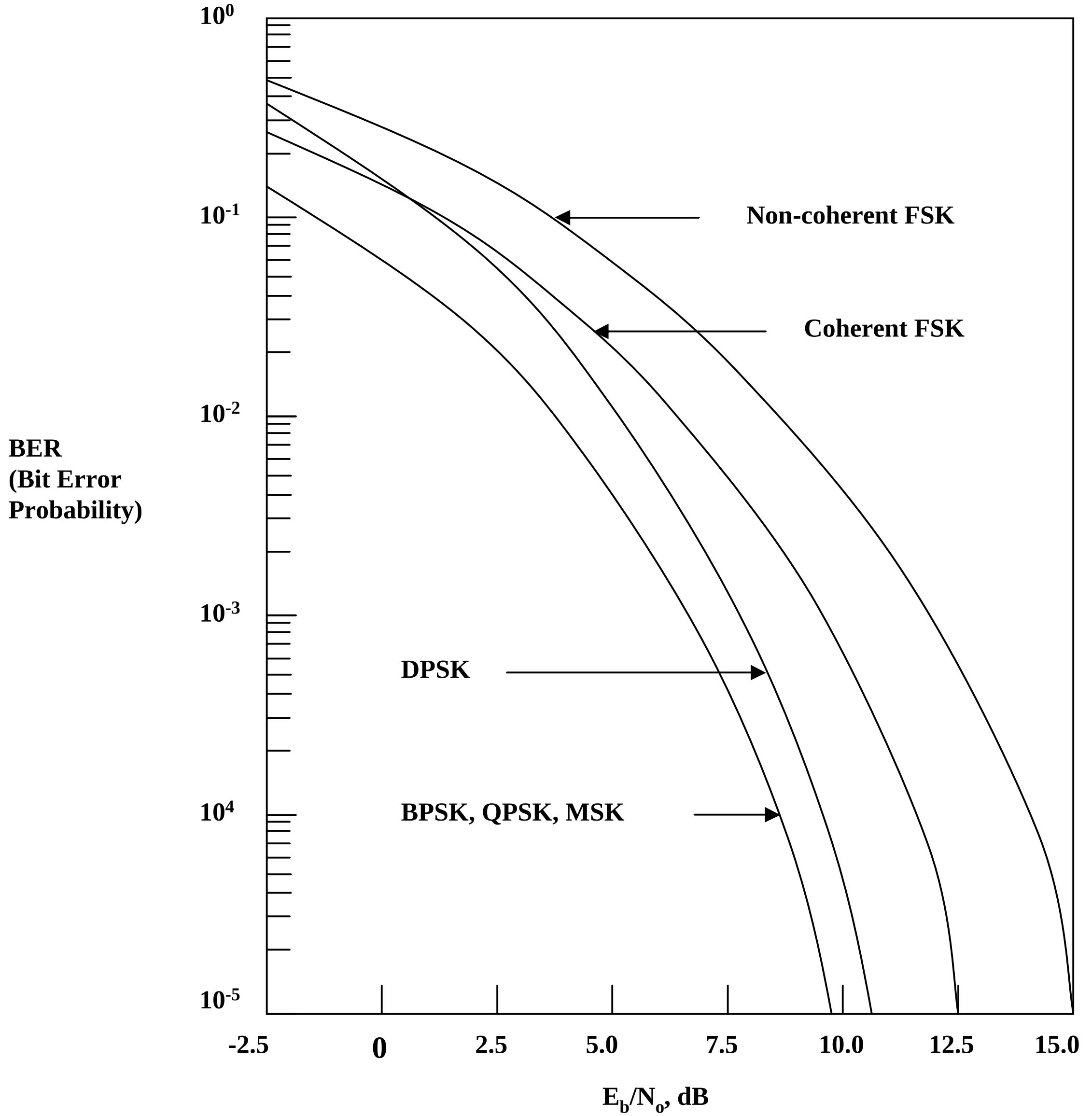


Figure 10.39 **Relationship between BER and E_b/N_o.**

o Error correction schemes

* Block encoding/decoding

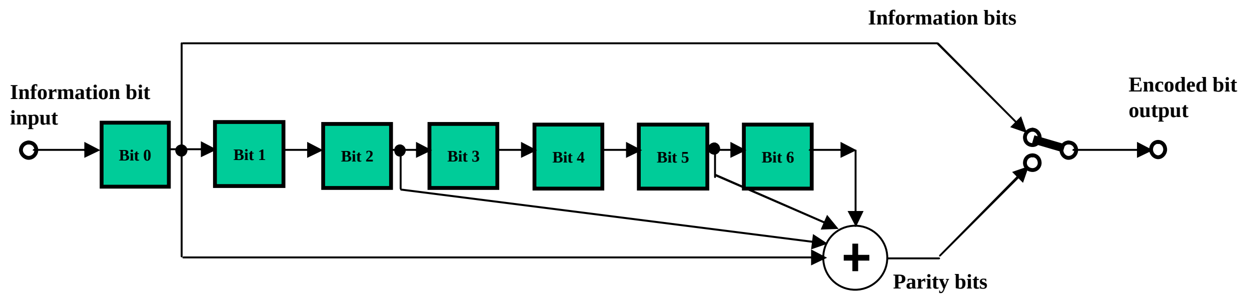
In error-correction block encoding, each block of *k* data bits is first encoded to a new block with *n* bits, in which *(n-k)* bits are redundant for the error correction. Of course,

$$n > k$$

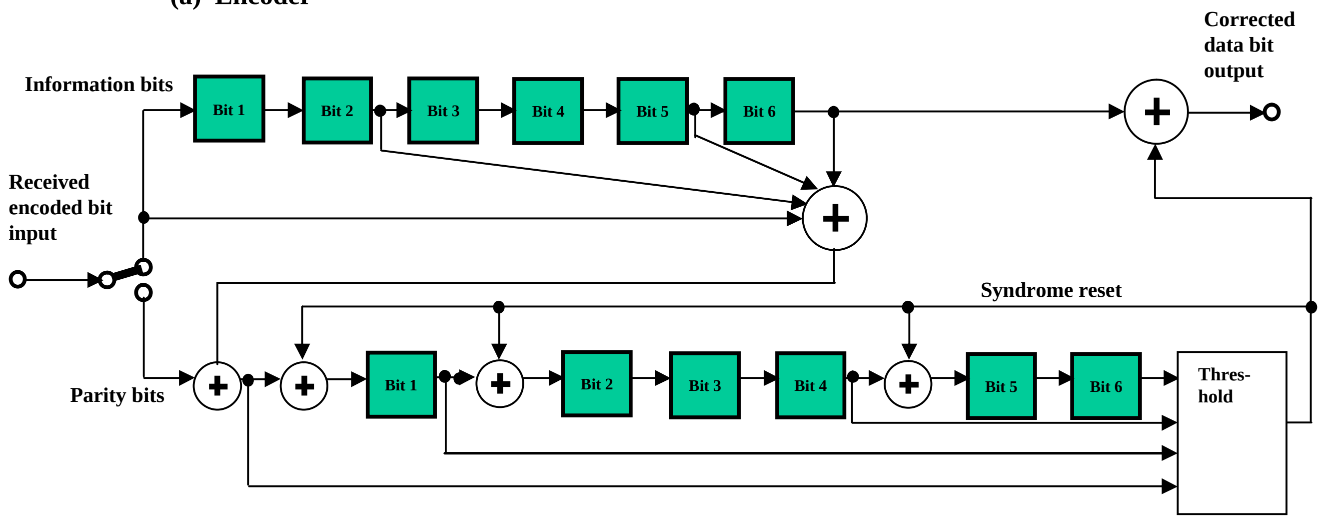
And, the code rate, r, is defined as

$$r = \frac{k}{n}$$

* Convolutionalencoding/decoding



(a) Encoder



(b) Decoder

Figure 10.40 A convolutionalcoding plan, r=1/2, k=7

* **Usually the main redundant bits are parity bits**
  In either block or convolutionalcoding ,

* **In convolutionalcoding, there are two kinds of redundant bits :**
  - **Parity bits;**
  - **CRC (Cyclic Redundant Check) bits.**

* **The advantages of block coding are**
  - **Simple;**
  - **Independent from block to block .**

* **The advantages of convolutionalcoding are**
  - **Shorter wordlength;**
  - **Special protection against burst error;**
  - **Lower word error probability.**

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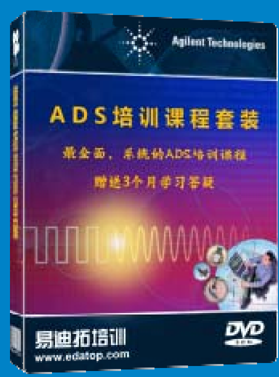

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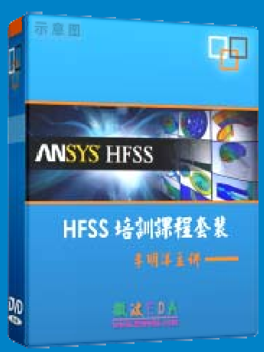

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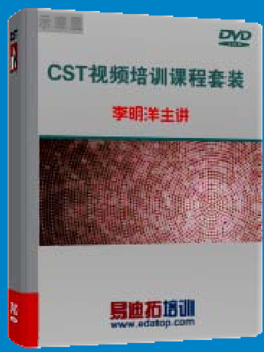

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